Quasi-Succinct Indices

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Inverted Indices

- The backbone of search engines (and more)
- Main problem: store a sequence of increasing integers in little space so to be able to pick the *i*-th integer / skip to the first integer larger than *b* in little time
- This is the classical rank/select problem
- For positions the problem is a bit more articulated (and complicated)

The Classical Solution

- Middle 80s/start of 90s (apparently depends on who you talk to)
- Turn the sequence $x_0, x_1, x_2, \ldots$ into *gaps* $x_0, x_1 - x_0, x_2 - x_1, \ldots$
- Hope that the numbers will be small and well (predictably) distributed
- Use some instantaneous code to store the gaps

Lot Of Research

- Zillions of different codes and kinds of codes
- Problem: sequential decoding easy, rank and selection *very inefficient*
- Solution: various kind of *skip tables* that make it possible to “jump” in the middle of the gap sequence
- In retrospective, it looks a little bit contrived, doesn’t it?

Why Gaps?

- Maybe we can approach the problem in a completely different way
- Maybe gaps were *not a good idea in the first place*
- Maybe there are nice, efficient ways of store sequences of integers that do not require gaps
- So, back (1975!) to the future (now)!

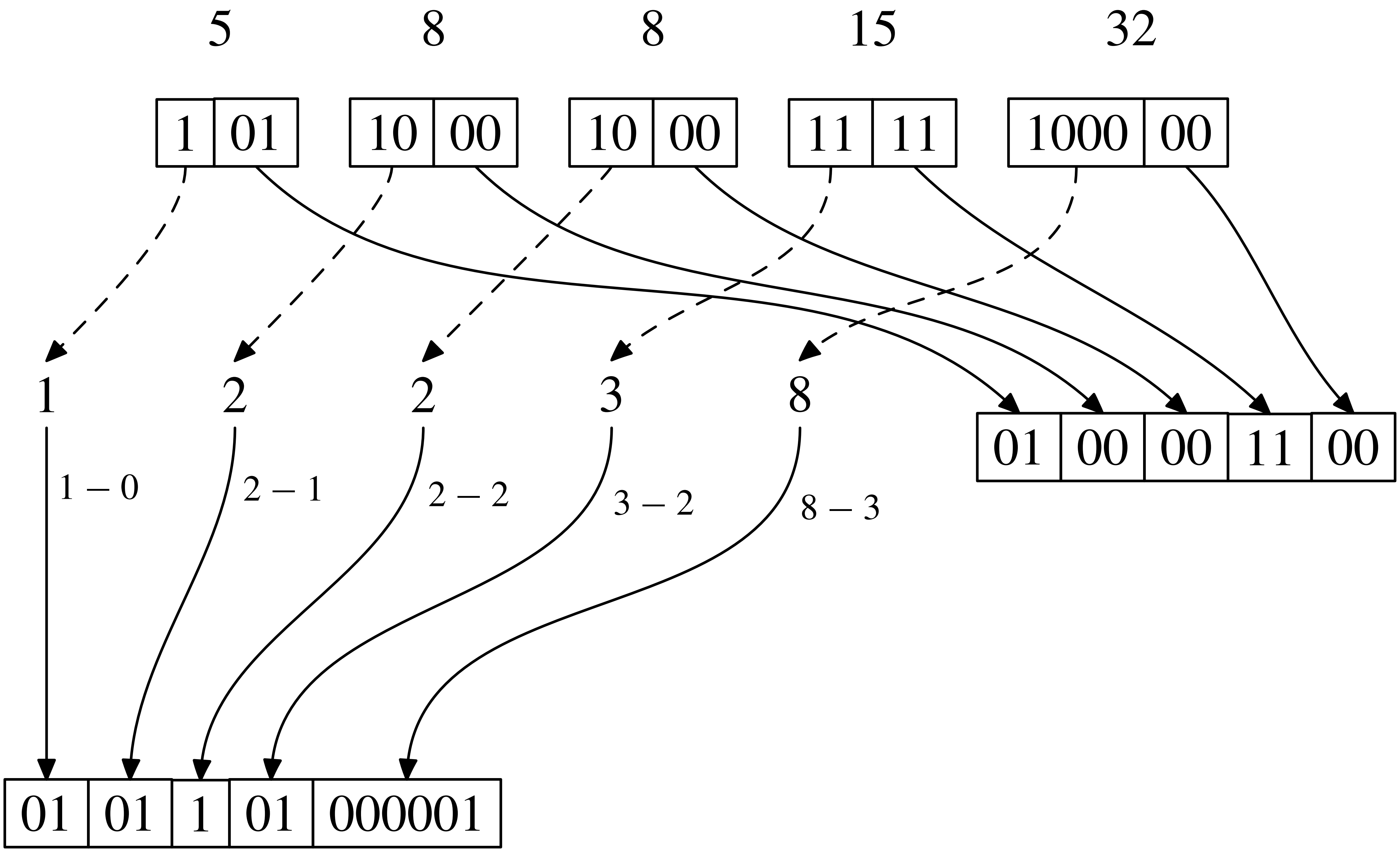
Elias-Fano Representation

- Elias developed in 1975 a quasi-succinct representation for monotone sequences (JACM); Fano discusses it in a report
- At that time, probably no more than a curiosity
- (My 2€¢: should be taught in the first year of any CS curriculum)
- We're going to revive the idea

High Bits/Low Bits

- Given n and u we have a monotone sequence

  $0 \leq x_0, x_1, x_2, \ldots, x_{n-1} < u$
- Store the lower $\ell = \log(u / n)$ bits explicitly
- Store the upper bits as a sequence of unary coded gaps (0^k1 represents k)
- We use at most $2 + \log(u / n)$ bits per element
- Close to the succinct bound: quasi-succinct!
- (Less than half a bit away, as Elias proves)



$5, 8, 8, 15, 32 < u = 36, \ell = 2$

Why Is This Any Good?

- Almost optimal space usage
- Distribution-free
- Reading sequentially requires very few logical operations (you might be surprised)
- Restrict the rank/selection problem to a nice ~$2n$ bits array with half zeroes, half ones
- It's beautiful :-)
- So, what about rank/select?

Looking up (Selection)

- Suppose you want to get the *k*-th element quickly
- Just scan the upper bits, one word at a time, doing population counting (one clock)
- Cost of searching: 100ps/element (yes, that's picoseconds) per element on an i7 @ 3.4GHz
- When you get to the right word, complete sequentially and pick the lower bits

Searching (Ranking)

- It's exactly the same: only, you count *zeroes*
- Zeroes tells you how much the *upper bits* are increasing, which is the important thing
- Just skip $b >> \ell$ *upper zeroes* and complete sequentially
- Due to the balance between ones and zeroes, on average always 100ps per element (this must be made more precise, see the paper)

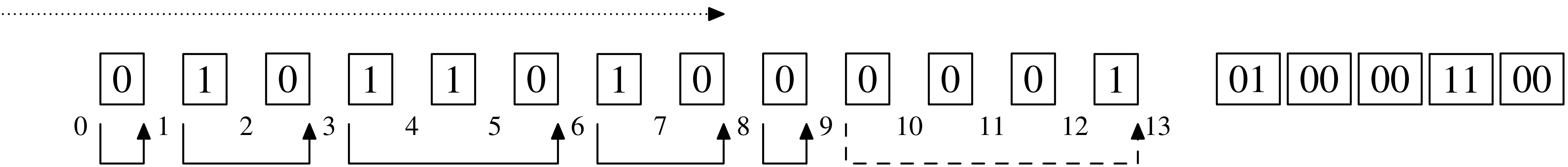
“Complete Sequentially”?

- Not really
- There are *broadword algorithms for selection* (I wrote the first one in 2007; improved later by Simon Gog)
- Fixed number of operations to skip *k* unary codes
- Final phase at ~500ps/element

```
int select_in_word( const uint64_t x, const int k ) {
   uint64_t byte_sums = x - ((x & 0xaaaaaaaaaaaaaaaaULL) >> 1);
   byte_sums = (byte_sums & 0x3333333333333333ULL) + ((byte_sums >> 2)
            & 0x3333333333333333ULL);
   byte_sums = (byte_sums + (byte_sums >> 4)) & 0x0f0f0f0f0f0f0f0fULL;
   byte_sums *= 0x0101010101010101ULL;
   const uint64_t k_step_8 = k * 0x0101010101010101ULL;
   const int place = ((((k_step_8 | 0x8080808080808080ULL) - byte_sums)
            & 0x8080808080808080ULL) >> 7) * 0x0101010101010101ULL >> 53 & ~0x7;
   return place + select_in_byte[x >> place & 0xFF |
            k - ((byte_sums << 8) >> place & 0xFF) << 8];
}
```

Not Fast enough?

- Fix a quantum q (I use 256)
- Store in a table the position of each q-th zero, or q-th one
- Go there in constant time and search from there
- On average, again constant time because of the balance between zeroes and ones
- Extreme locality: one memory access per skip



- $5, 8, 8, 15, 32 \leq u = 36, \ell = 2$
- We to skip to 22, so we skip $22 >> \ell = 5$ zeroes
- We getting to position 9, so we are in the middle of the unary code associated with the element of index 9 - 5 = 4
- A unary-code read (the dashed arrow) returns 3
- We now know that the upper bits of the current element (of index 4) are 3 + 5 = 8
- Since the block of lower bits of index 4 is zero, we return 32
- If we have skip pointers with q=4, we can start from the dotted arrow

Enough of Fun with Bits

- We want to store an inverted index
- There are document *pointers*, *counts* and *positions*
- For pointers we obviously use a quasi-succinct list with skips
- Counts? Positions?
- Important: we can store *strictly* monotone sequences quasi-succinctly by storing $x_i - i$!

Using Duality Perversely

- Instead of storing counts $c_0, c_1, c_2, \ldots$, we store their *prefix sums* (a.k.a. *cumulative function*) $c_0, c_0 + c_1, c_0 + c_1 + c_2, \ldots$
- Instead of storing positions, we store the *prefix sums of their gaps*
- Main combinatorial idea (probably, the only actual idea in the paper): *the prefix sum of counts is the indexing function for positions*

Fast & Compact

- Decoding speed faster than other approaches (but not for counts!)
- Compression *definitely* better than other approaches, even for the smallest lists, except for very slow stuff like Golomb
- Locality of access *definitely* better than other appoaches
- Scalability (in theory and practice) better than other approaches

What Now?

- Let's improve this, e.g., *better implementations*
- There's decades of engineering and optimization on gaps, nothing on this, yet it is faster and compresses better!
- Beautiful code by Philip Pronin (Facebook) on GitHub:

```
int64_t get_next_upper_bits() {
  while( word == 0 ) word = upper_bits[ ++curr ];
  const int64_t upper_bits = curr * 64 +
                     __builtin_ctzll( word ) - index++;
  word &= word - 1;
  return upper_bits;
}
```

Recent news

- Best paper at SIGIR 2014: Giuseppe Ottaviano and Rossano Venturini for “Partitioned Elias–Fano indexes”.
- They break optimally a list into a small number of chunks so that each chunk compresses better
- Makes Elias–Fano somewhat “distribution-aware”

Try It!

- On MG4J: http://mg4j.di.unimi.it/
- Facebook: https://github.com/facebook/folly/
- You might find your own application
- WebGraph has an EFGraph implementation
- Questions?