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- If $c > 2.09$ the random graph with edges $g(x)$—$h(x)$ is almost surely *acyclic* as $n \to \infty$
- We *peel* the graph removing leaves iteratively
- Every edge is associated with the leaf that caused its removal

An example

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$f:X \to [16]$

$z \mapsto 0$

$a \mapsto 3$

$p \mapsto 11$

$u \mapsto 7$

$c \mapsto 2$

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$f:X \to [16]$	$g:U \to [7]$
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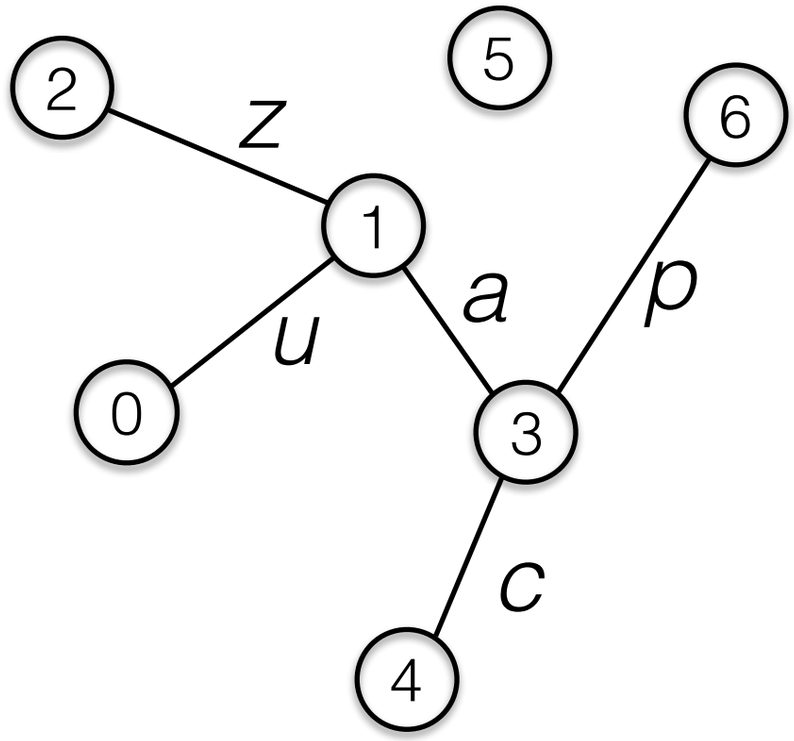

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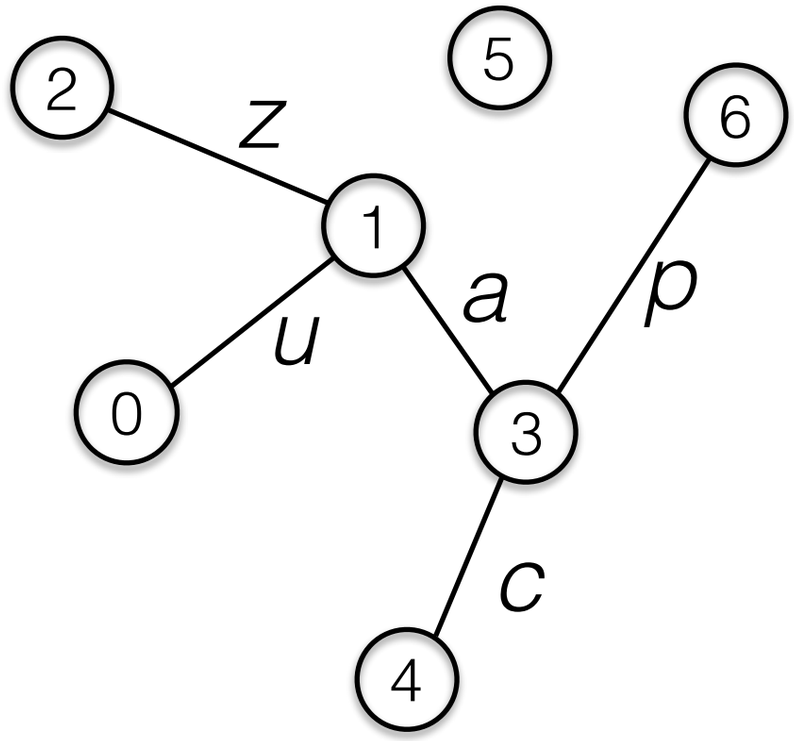

$$v_1 \oplus \boldsymbol{v_0} = f(u) = 7$$

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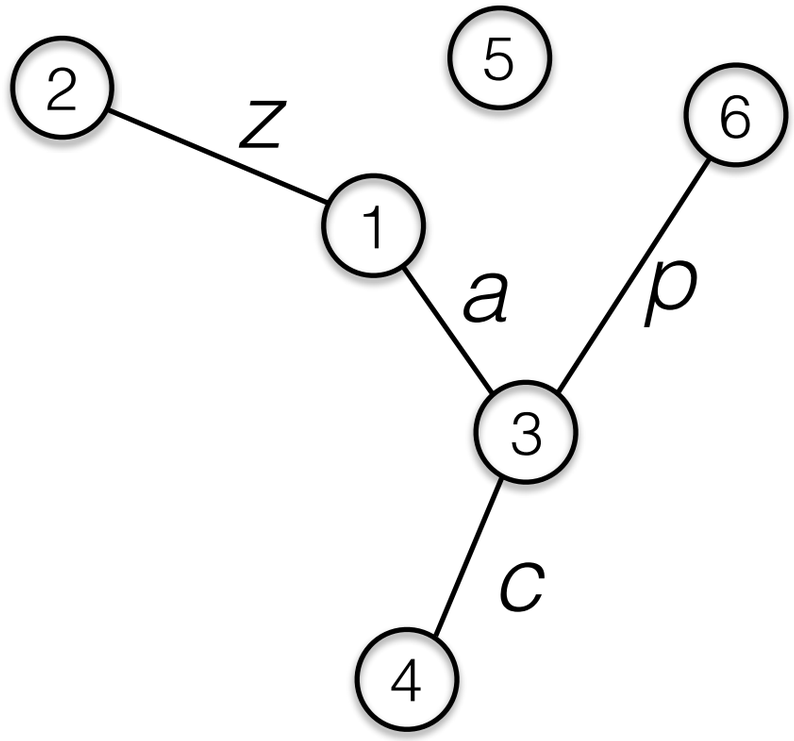

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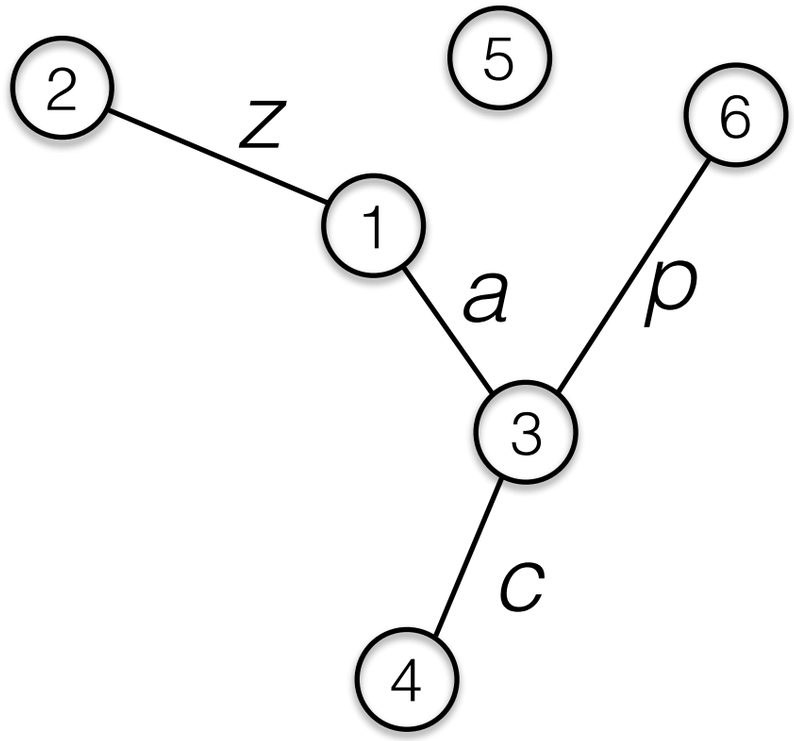

$$\boldsymbol{v_4} \oplus v_3 = f(c) = 2$$

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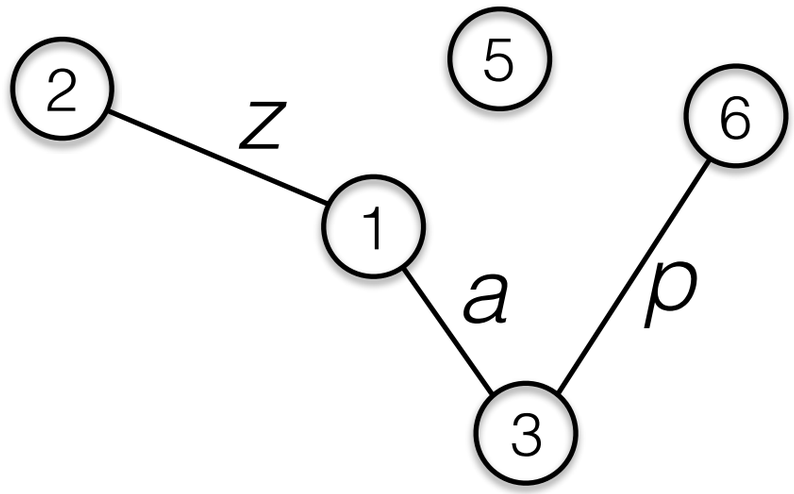

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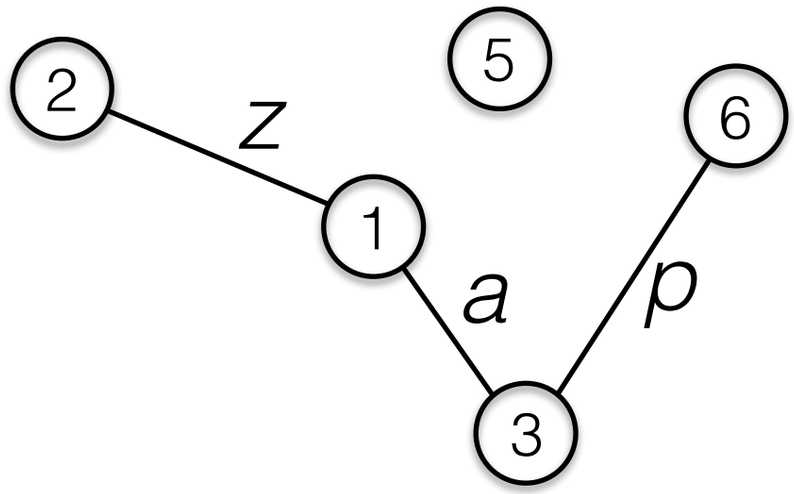

$$v_3 \oplus \boldsymbol{v_6} = f(p) = 11$$

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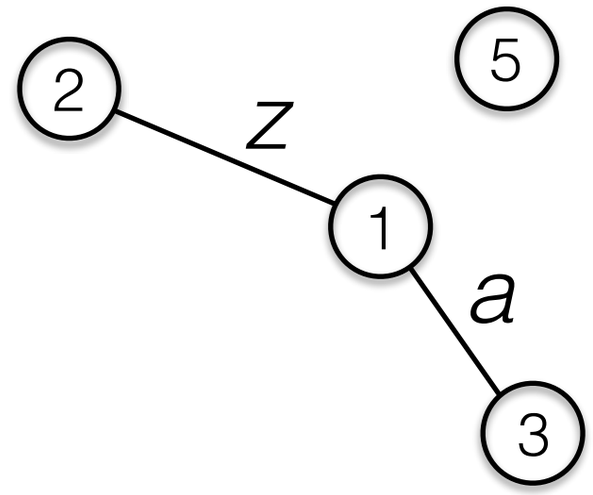

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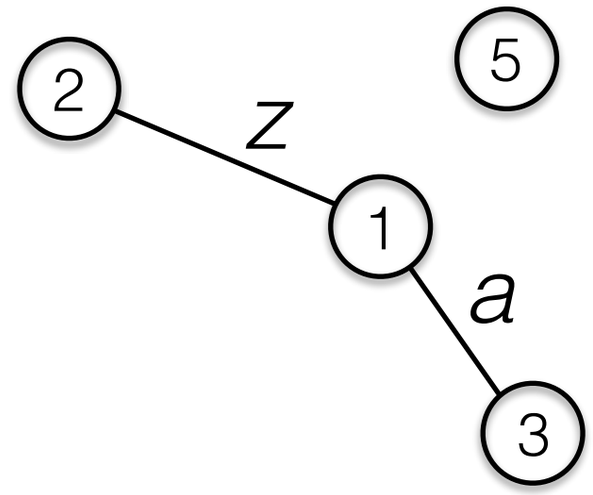

$$v_1 \oplus \boldsymbol{v_2} = f(z) = 0$$

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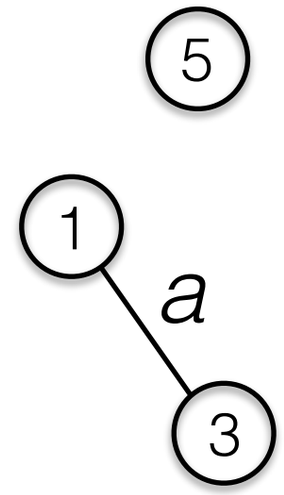

$$v_1 \oplus \boldsymbol{v_2} = f(z) = 0$$

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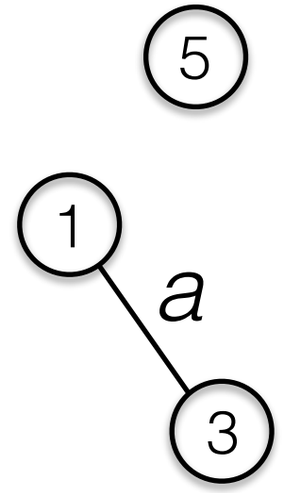

$$v_3 \oplus \boldsymbol{v_1} = f(a) = 3$$

$$v_1 \oplus \boldsymbol{v_2} = f(z) = 0$$

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5

3

$v_3 \oplus \boldsymbol{v_1} = f(a) = 3$

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$v_3 = 0,\ \boldsymbol{v_1} = 3$

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(5)

(3)

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$v_3 = 0,\ \boldsymbol{v_1} = 3$

$\boldsymbol{v_2} = v_1 \oplus 0 = 3$

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$\boldsymbol{v_4} \oplus v_3 = f(c) = 2$	$\boldsymbol{v_4} = v_3 \oplus 2 = 2$
$v_1 \oplus \boldsymbol{v_0} = f(u) = 7$	$\boldsymbol{v_0} = v_1 \oplus 7 = 4$