

# Multirelational Semantics for Extended Entity–Relationship Schemata With Applications

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# Motivation

# ERW

- A tool bridging conceptual modelling with web applications

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- Starting from any schema, generate a web-based content manager

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- Starting from any schema, generate a web-based content manager
- Wide range of EER schema features supported
- Need for validation
- Concept similar to WebML (Ceri, Fraternali & Bongio WWW 2000) but different scope and features

# ERW

Book: Algebraische Grundlagen der Informatik, by K.-U. Witt - Galeon

OK Apply Apply & Clone Cancel

Title:  Author:

Publisher:  ISBN Code:

Year:

Description:

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Daniel Abbw-Jackson (Long term, 01/02/2002-)  
Emile H. L. Aarts (Long term, 02/05/2001-)  
Jan van den Heuvel (Long term, 02/03/2002-03/04/2002)

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Jonathan Aaronson  
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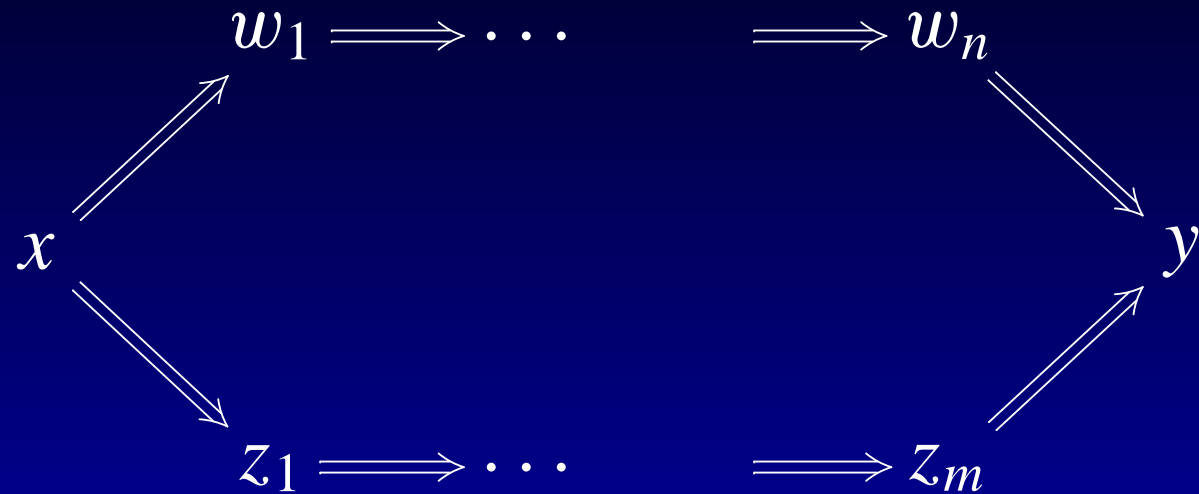
- *Acyclic typing*: type hierarchies must be DAGs (Directed Acyclic Graphs)
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- *No double ownership*: no schema instance contains two entities  $x$  and  $y$  such that  $y$  owns  $x$  along two distinct ownership paths

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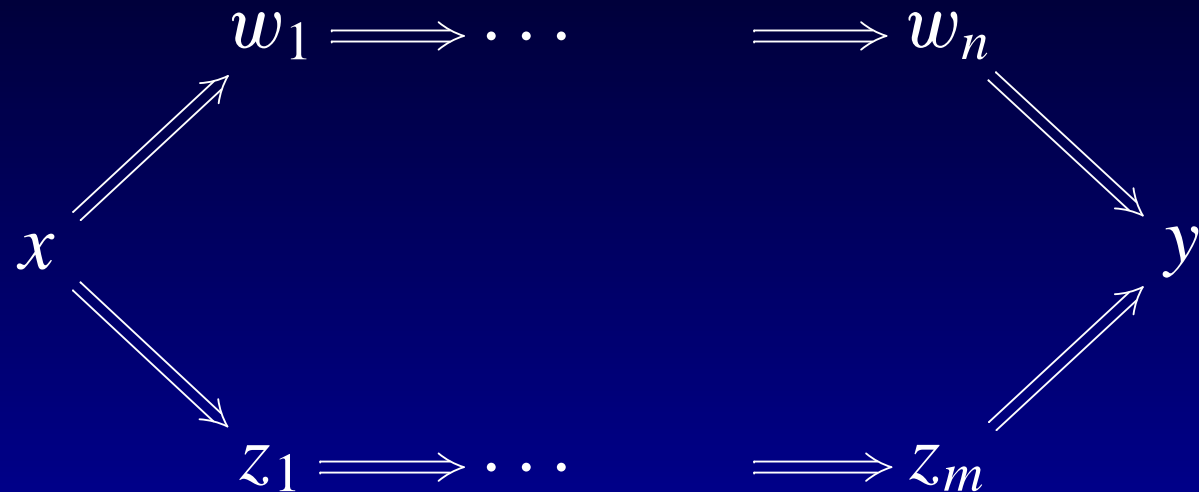
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- We concentrate on the third condition (the “application” in the title)

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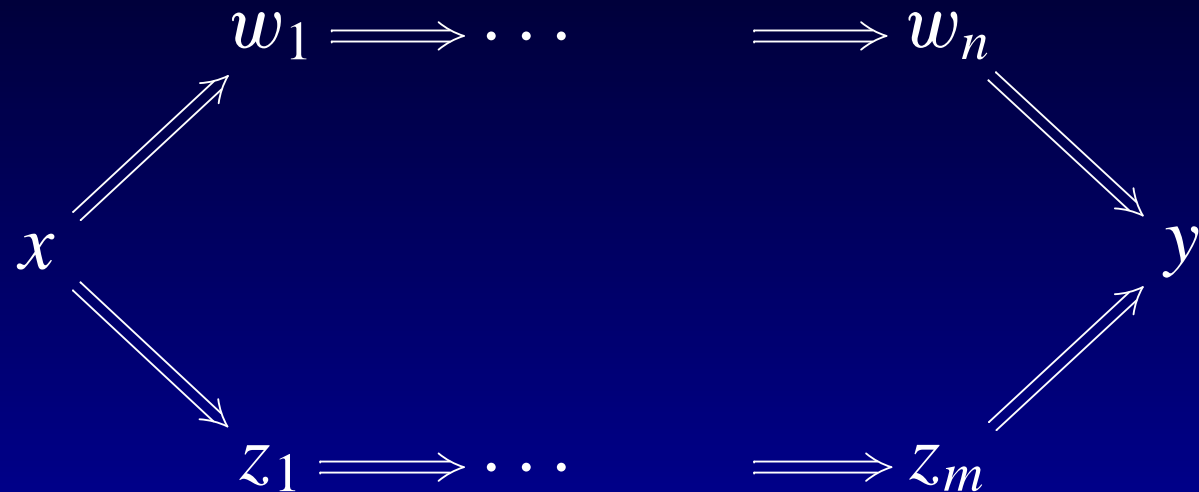


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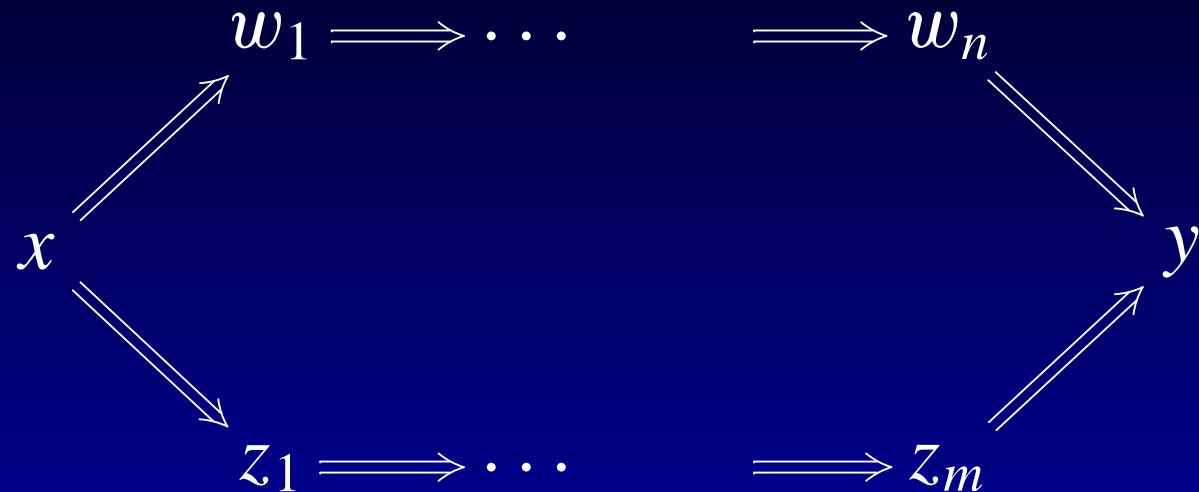


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- It is a *semantic* condition: it happens in a schema *instance*
- We are interested in rejecting *syntactically* the bad schemata
- You need a formal semantics, or things mess up (e.g., Balaban & Shoval ER 1999)

# The Typeless Case

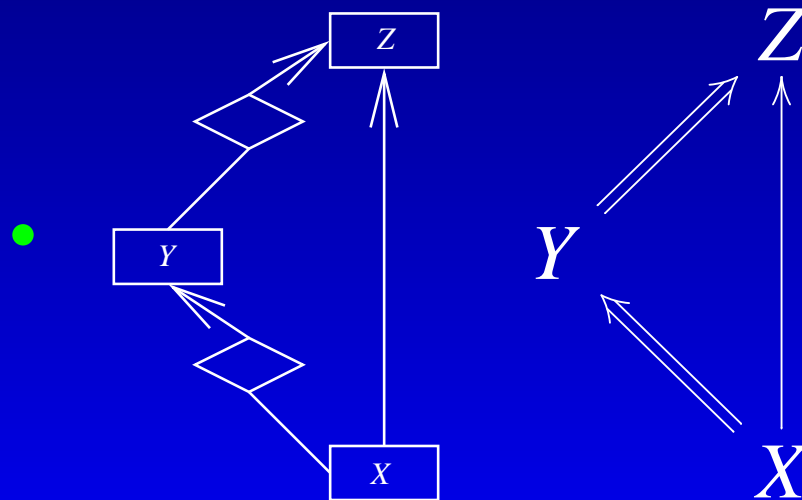
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- Collapse type hierarchies into points, and check for parallel paths
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- ...but none of the above works
- To prove a soundness and completeness result we need a formal, mathematical semantics (e.g., see Gogolla & Hohenstein 1991, Thalheim 2000,...)

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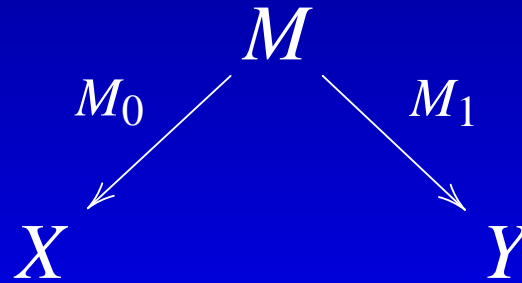
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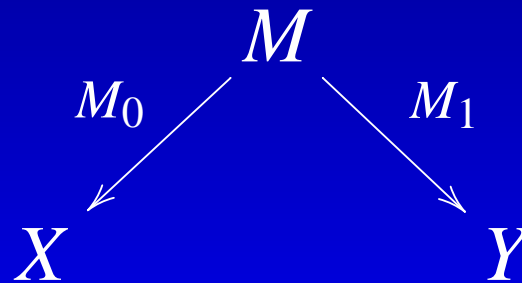
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- Provides ready-made composition etc.

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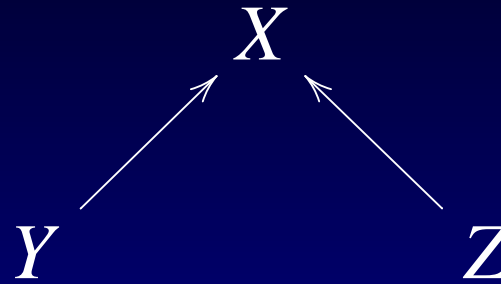
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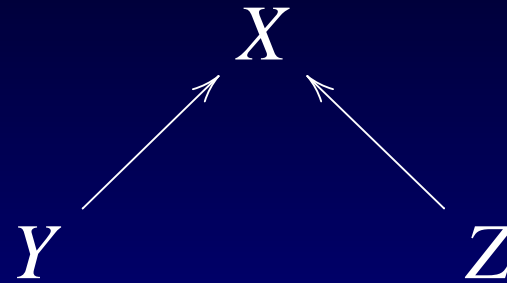
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- Not in general!

# Types (2)

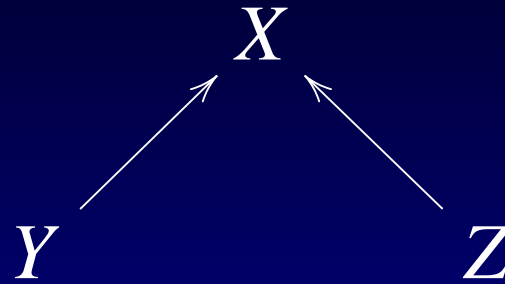


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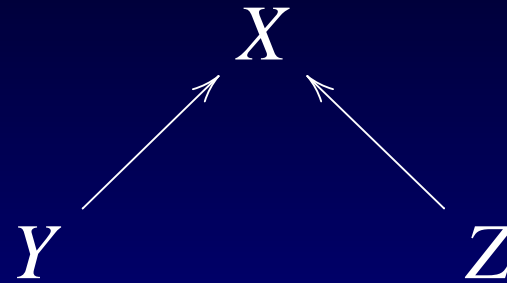
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- Conjunctive vs. disjunctive subtyping



# Conjunctive (or Free) Typing

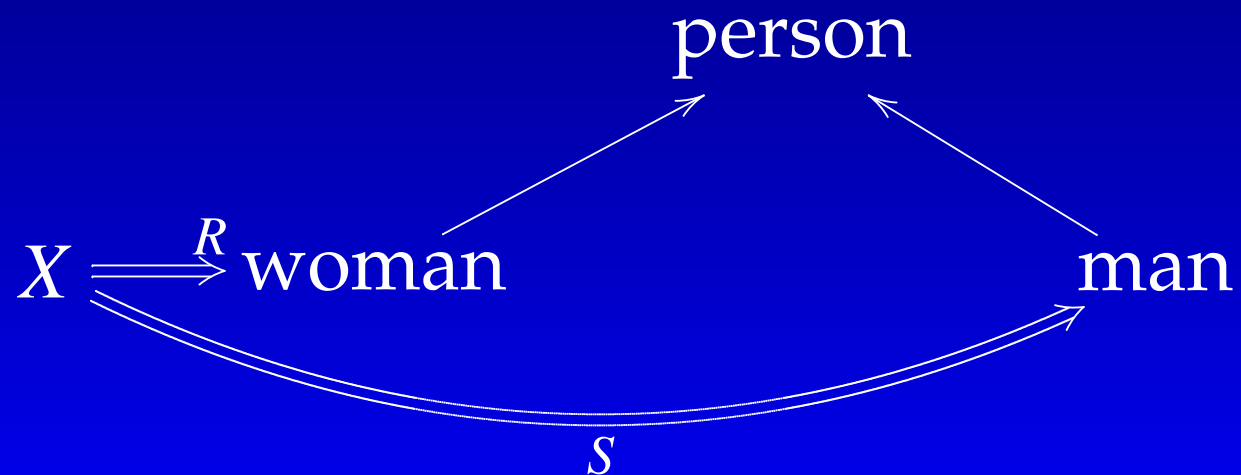
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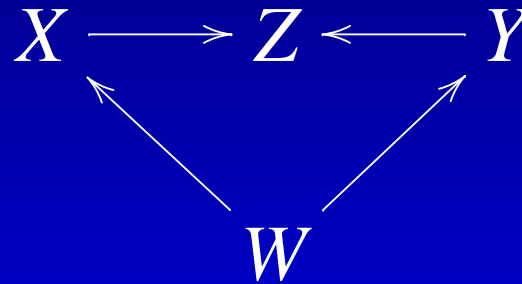


# Definite Typing

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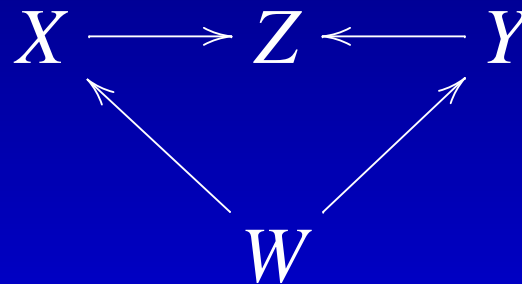
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- Mathematically, you need a *stability* condition on the semantic map (Berry 1978, see the paper)

# Definite Typing (2)

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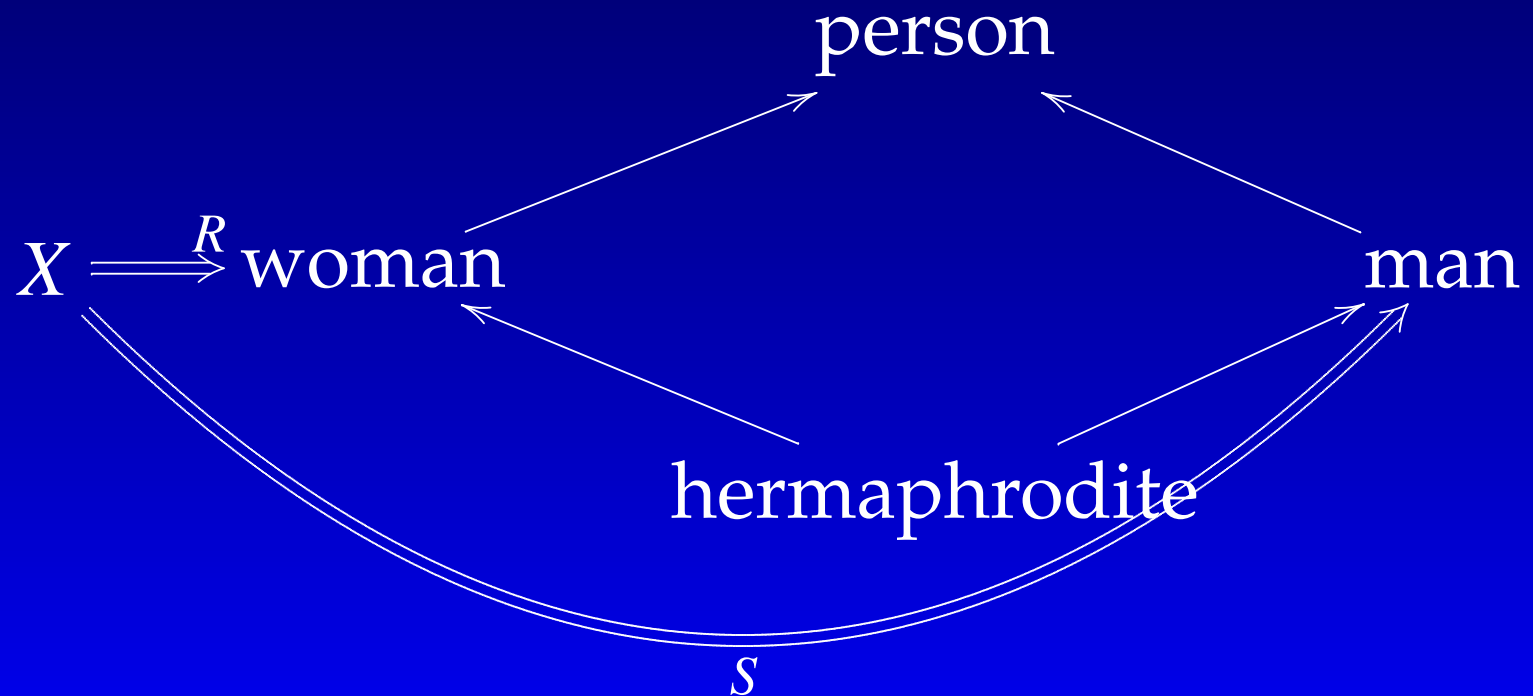
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# Validating Ownership

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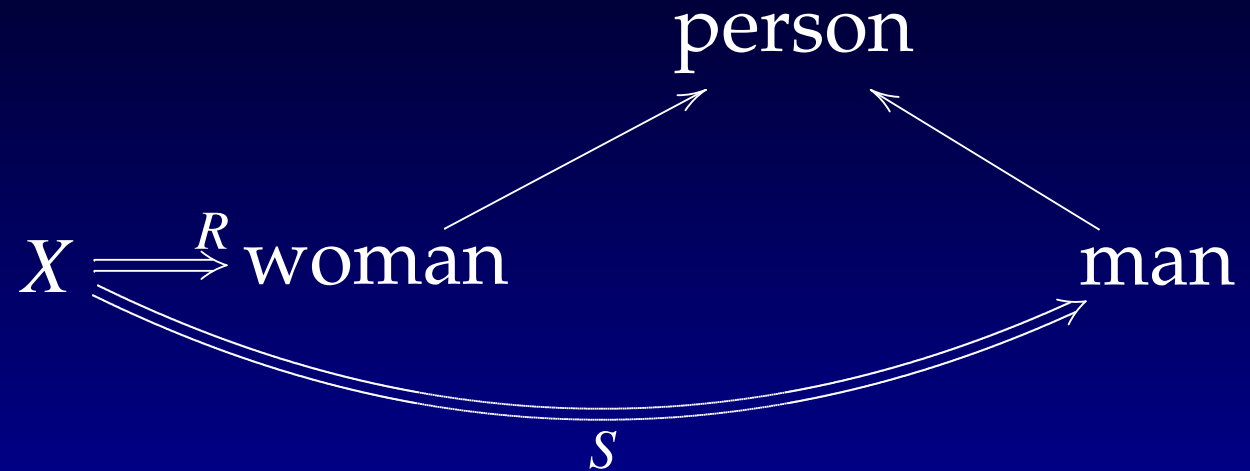
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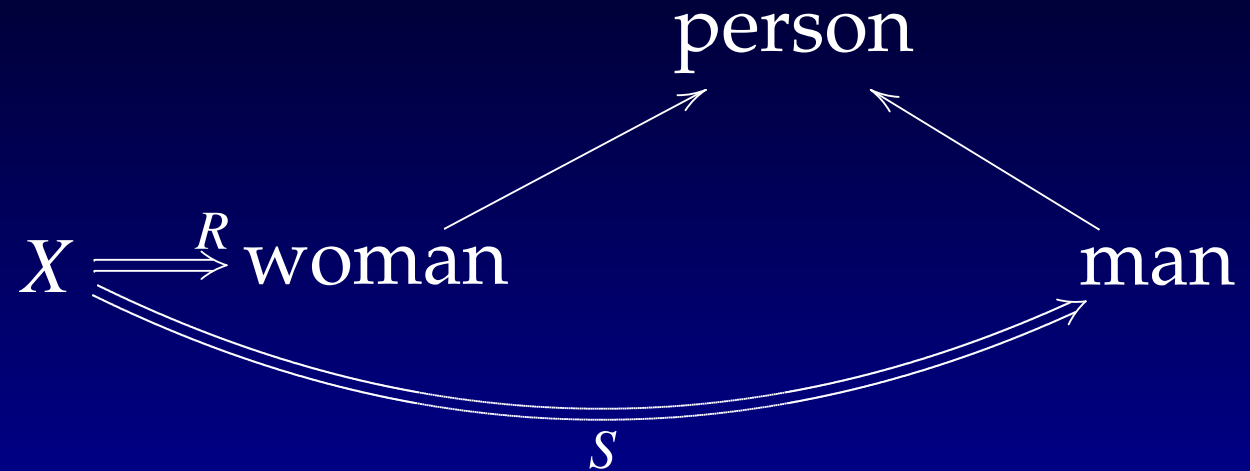
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- Can we just build a new graph with extended identification functions, and check that no pair of parallel paths exists?

# It May Work!



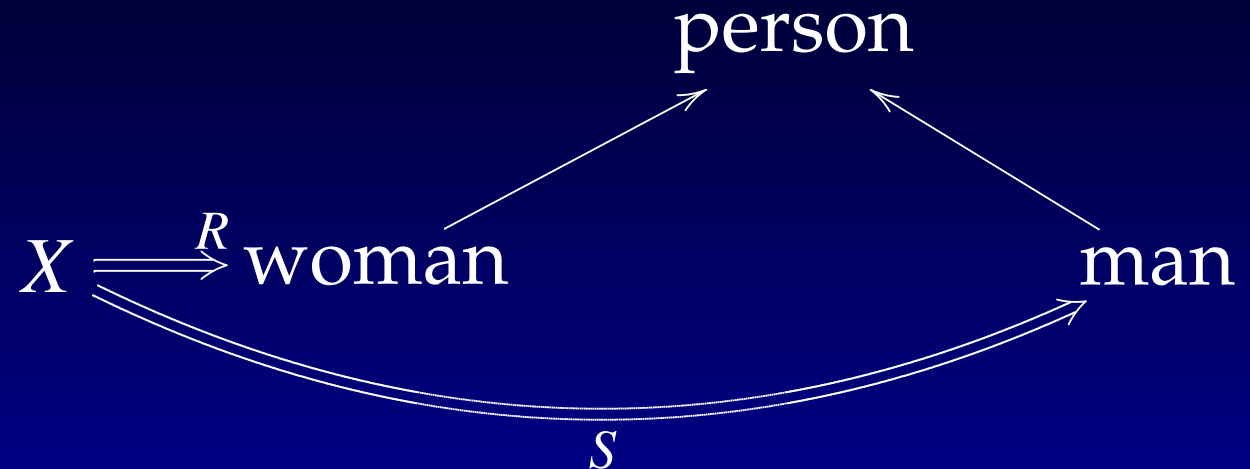
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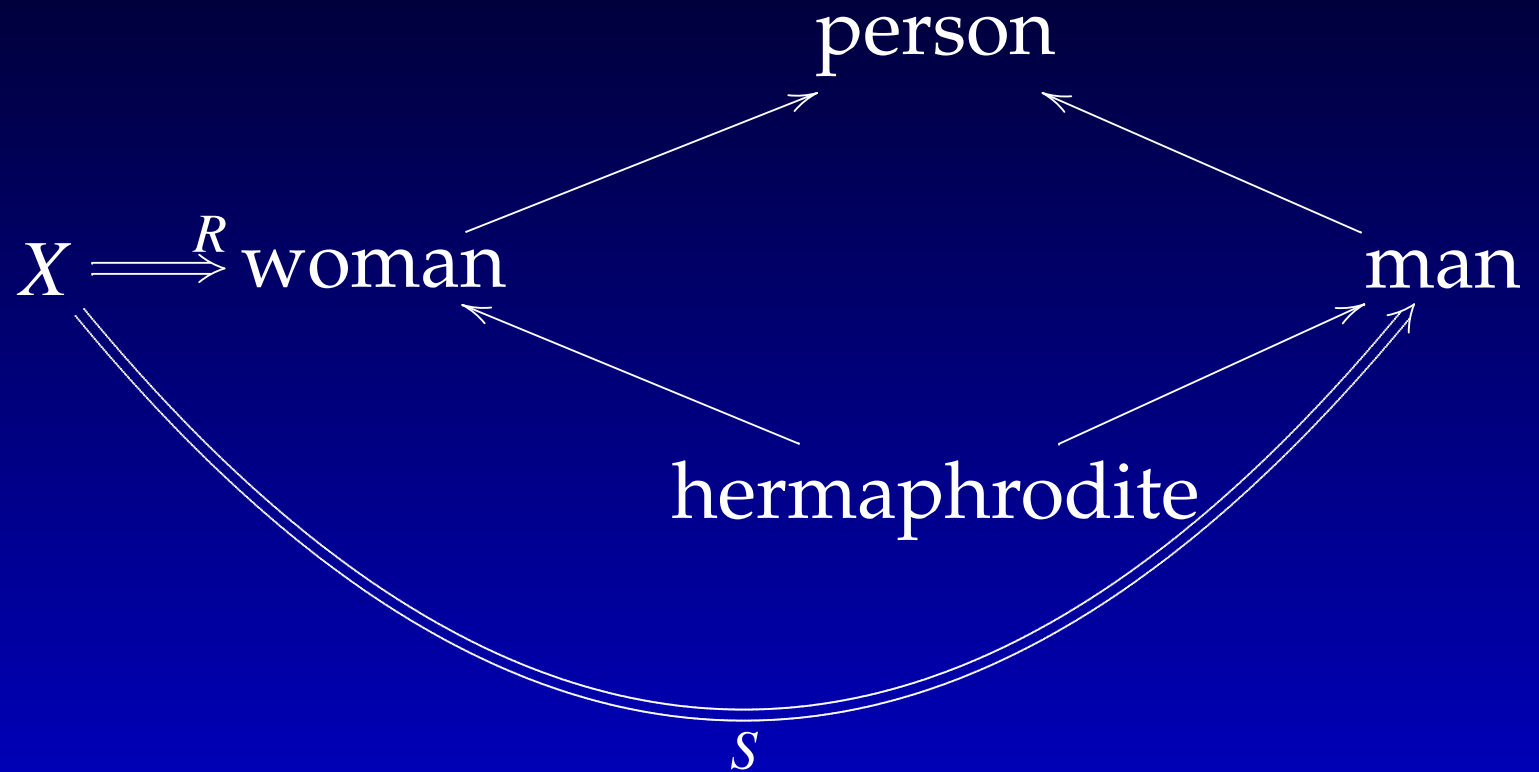
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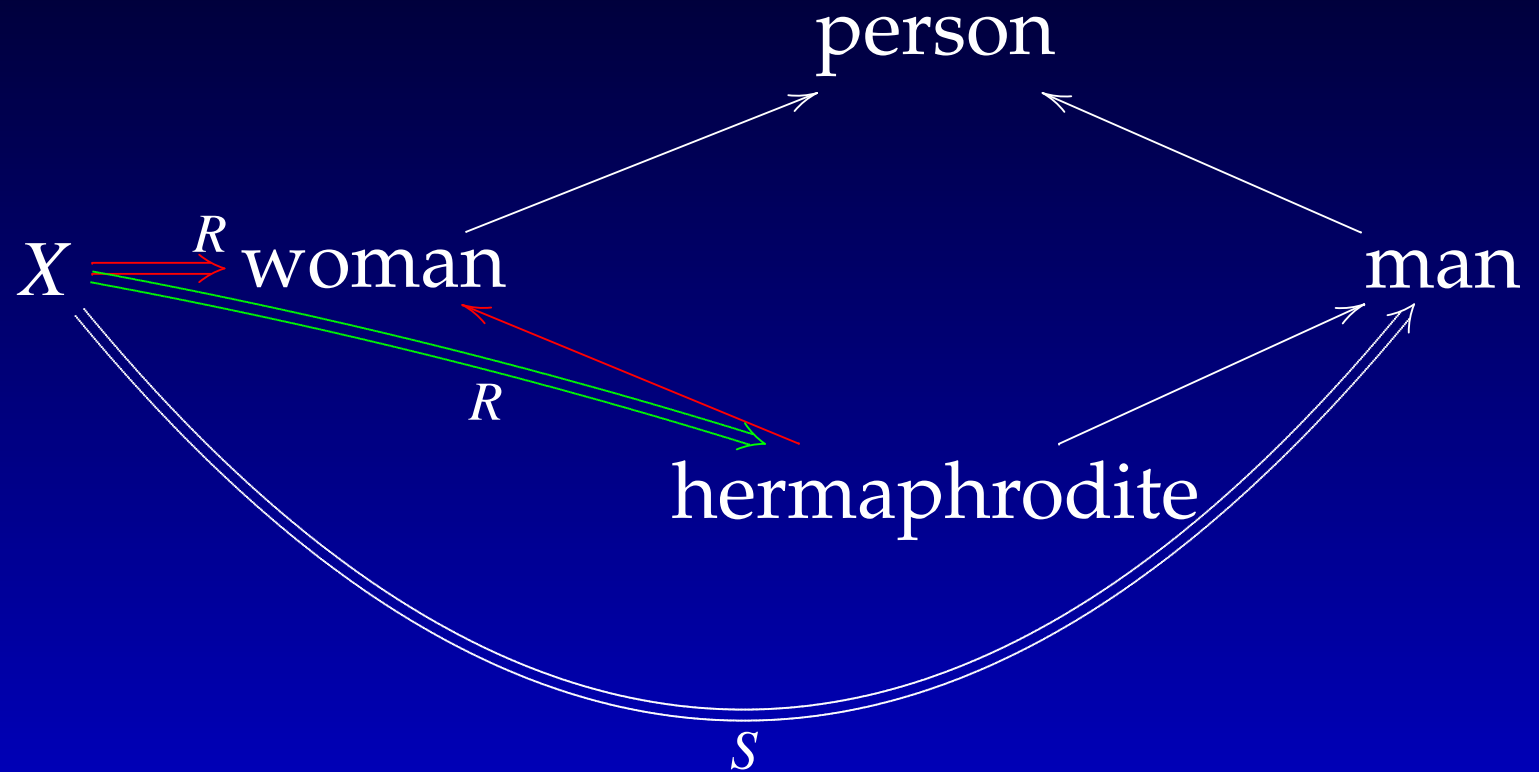
- No additional arcs
- The schema is valid



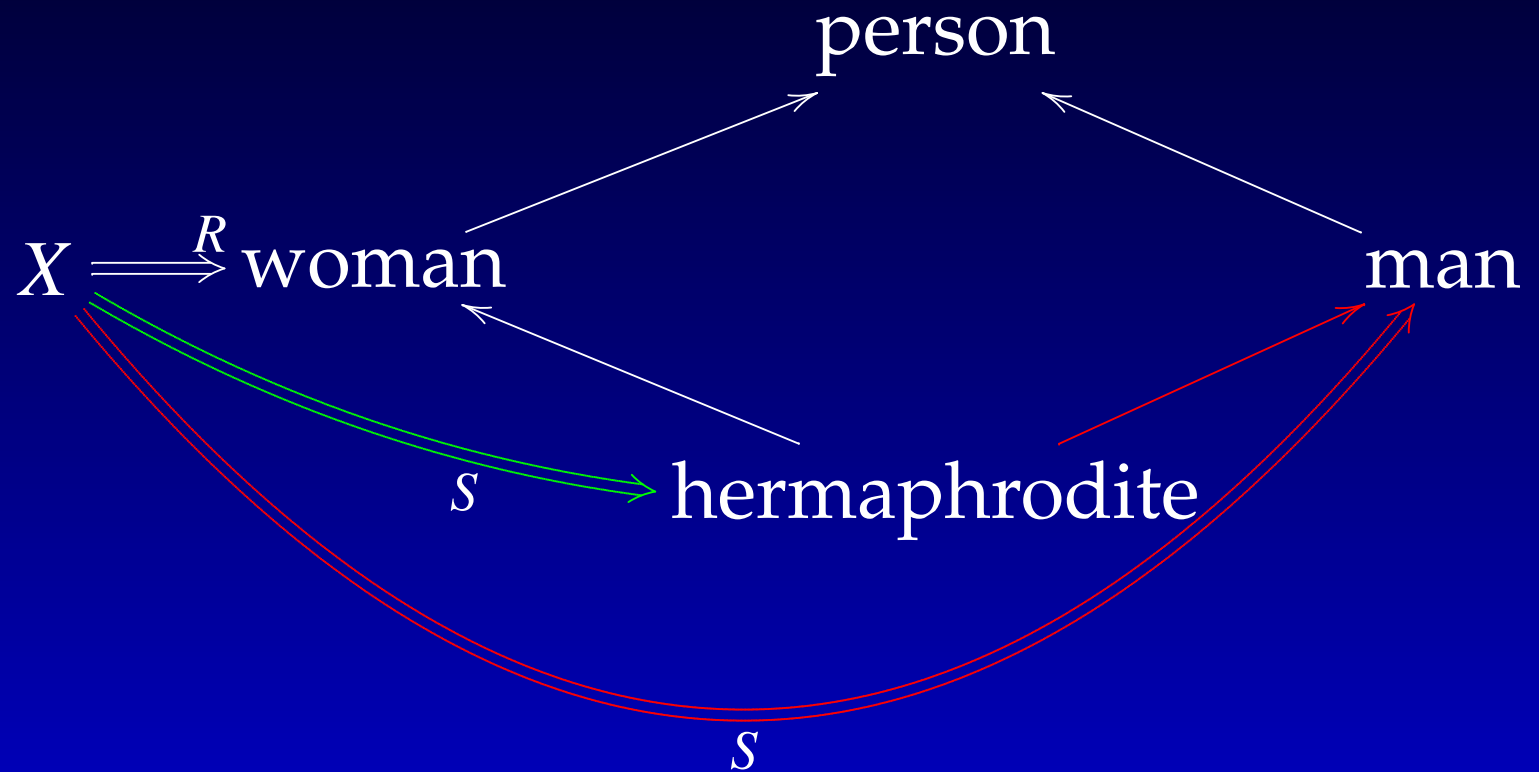
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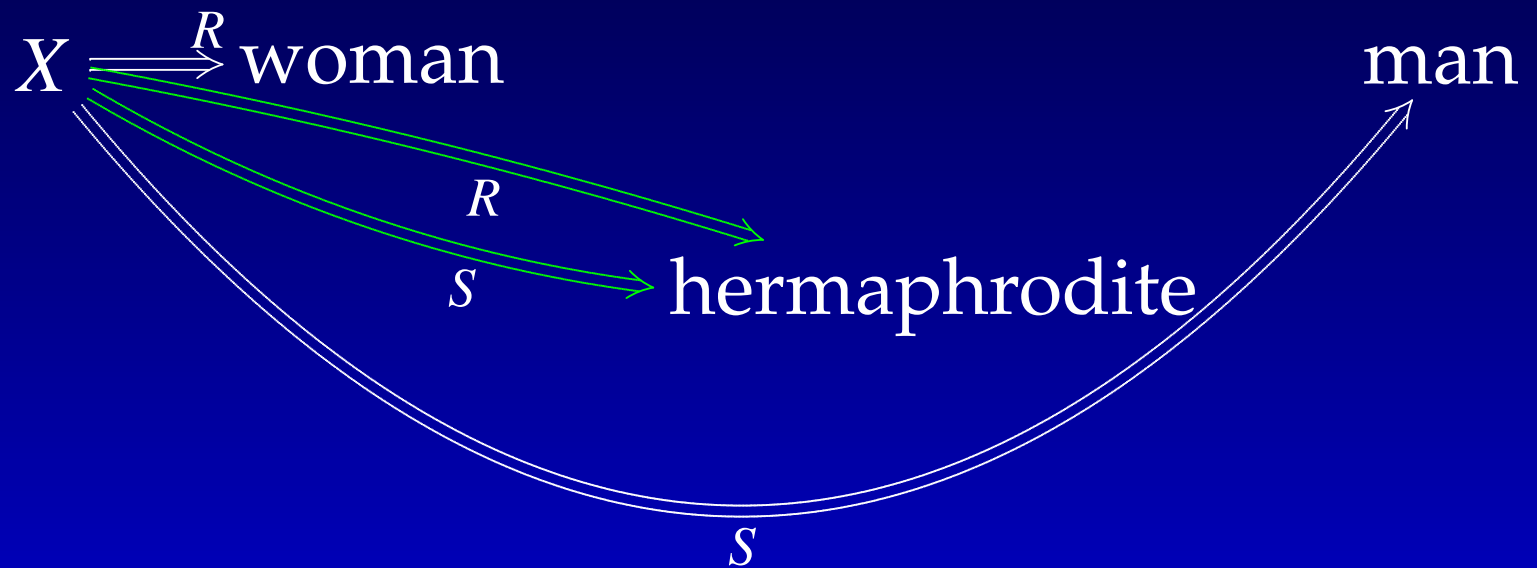


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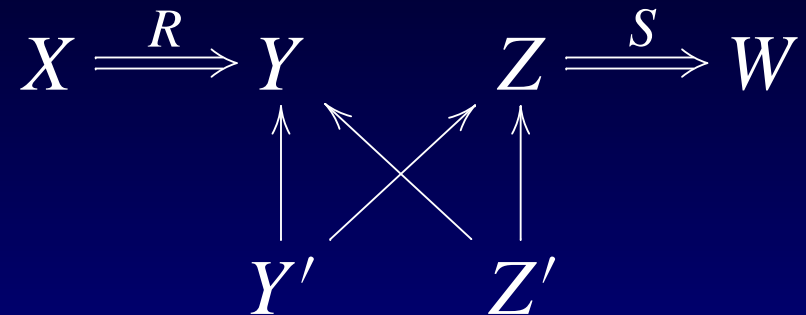


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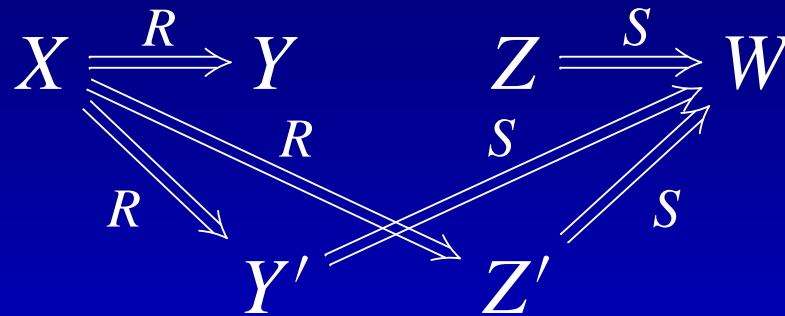
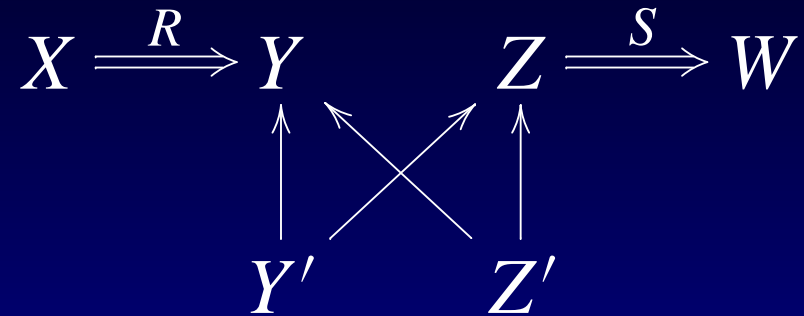
person



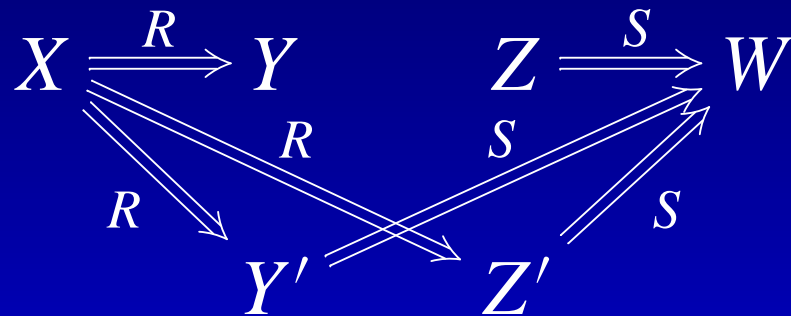
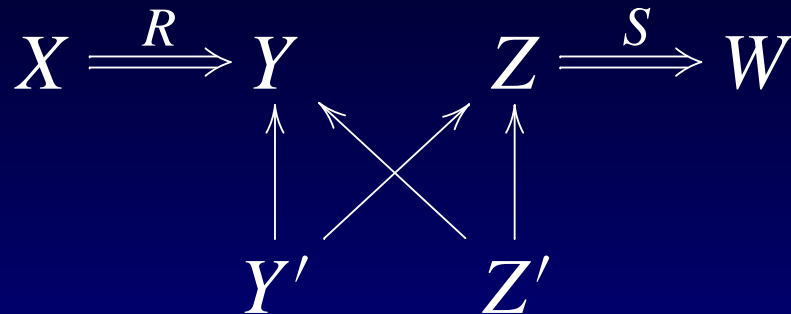
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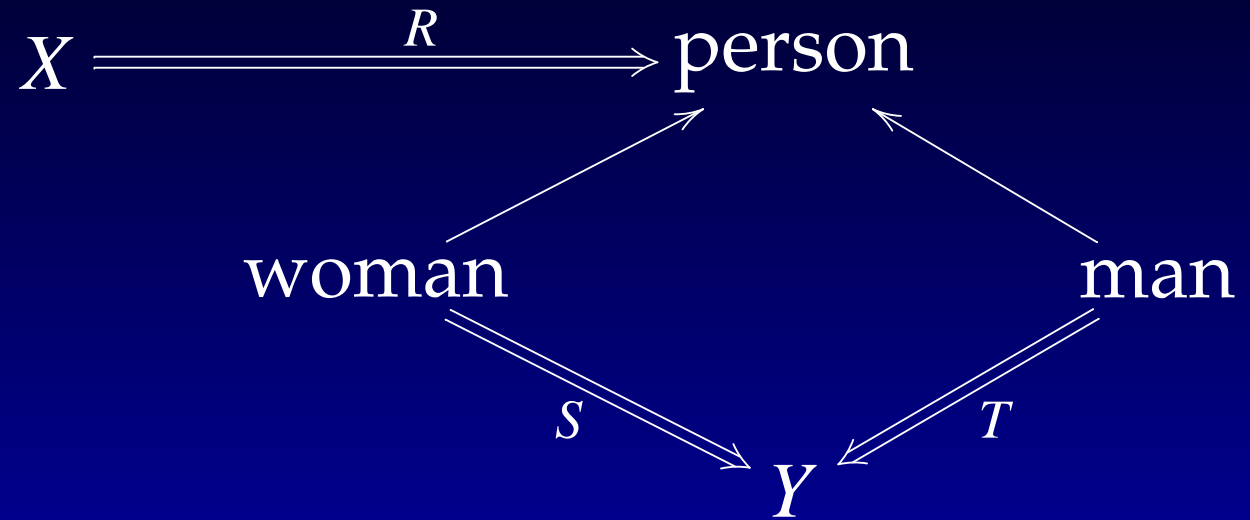


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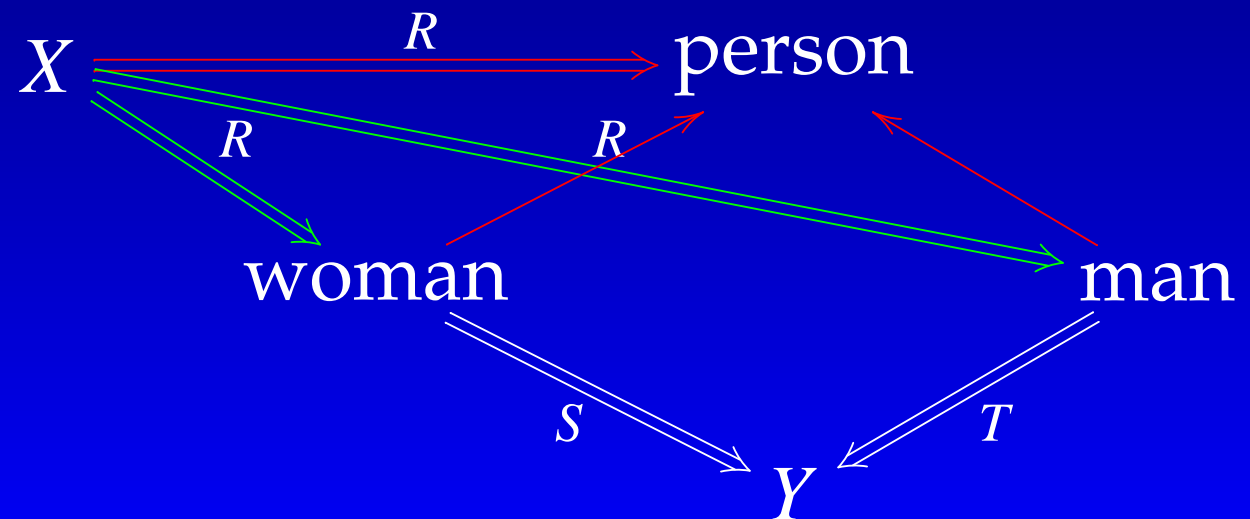
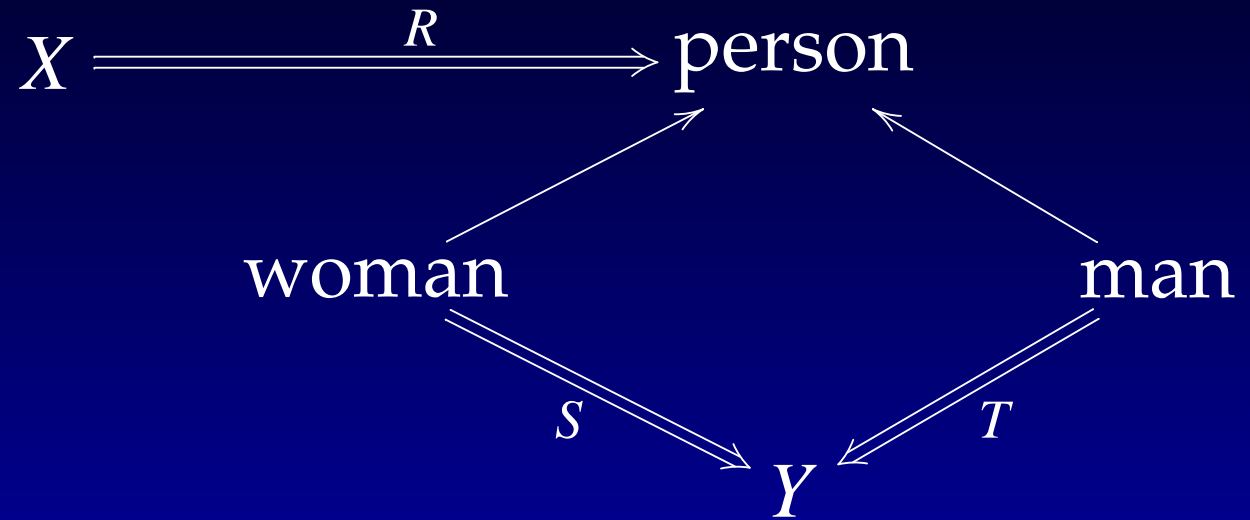
- Thus, not all pairs of parallel paths are bad. Maybe we should check that the sequence of labels (identification functions) is different...

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- Indeed, if double ownership is possible, then it must happen in such a way that *the first identification function is different*:

$$\begin{array}{ccc}
 x \xrightarrow{R} R(x) & \rightsquigarrow & R(x) \xrightarrow{S} y \\
 \begin{array}{c} \Downarrow R \\ R(x) \end{array} & & \begin{array}{c} \Downarrow T \\ z \end{array} \\
 \begin{array}{c} \Downarrow \\ w \end{array} & & \begin{array}{c} \Downarrow \\ w \end{array} \\
 R(x) \rightsquigarrow w & & z \rightsquigarrow w
 \end{array}$$

# A Polynomial Algorithm

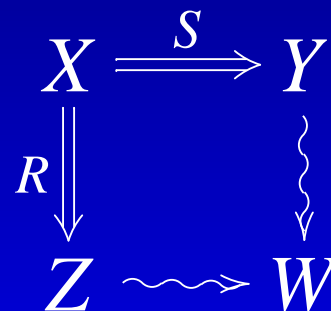
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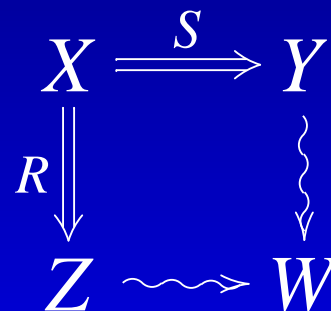
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- Everything here is polynomial (albeit of rather high degree)

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- With some care, this leads to an algorithm that is cubic in the entity types and quadratic in the identification functions, provided that there is an upper bound on the size of type hierarchies
- Essentially, it's cubic (works for thousands of entity types)

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- This is in line with common practise in SQL databases (rows in different tables use the same numerical identifiers)
- By making slightly stronger the stability condition, one can force all elements representing different entities to be distinct (“theoretician’s semantics”)
- The bicategorical composition of multirelation (pullback) is simply the join operation from relational algebra