Quasi-Succinct Indices

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ABSTRACT
Compressed inverted indices in use today are based on the idea of gap compression: documents pointers are stored in increasing order, and the gaps between successive document pointers are stored using suitable codes which represent smaller gaps using less bits. Additional data such as counts and positions is stored using similar techniques. A large body of research has been built in the last 30 years around gap compression, including theoretical modeling of the gap distribution, specialized instantaneous codes suitable for gap encoding, and ad hoc document reorderings which increase the efficiency of instantaneous codes. This paper proposes to represent an index using a different architecture based on quasi-succinct representation of monotone sequences. We show that, besides being theoretically elegant and simple, the new index provides expected constant-time operations, space savings, and, in practice, significant performance improvements on conjunctive, phrasal and proximity queries.

Categories and Subject Descriptors
H.3.1 [Information Storage and Retrieval]: Content Analysis and Indexing—Indexing methods

Keywords
Compressed indices; succinct data structures

1. INTRODUCTION
An inverted index over a collection of documents contains, for each term of the collection, the set of documents in which the term appears and additional information such as the number of occurrences of the term within each document, and possibly their positions. Inverted indices form the backbone of all modern search engines, and the existence of large document collections (typically, the web) has made the construction of efficient inverted indices ever more important.

Compression of inverted indices saves disk space, but more importantly also reduces disk and main memory accesses [7], resulting in faster evaluation. We refer the reader to the book by Mann, Raghavan and Schütze [19] and to the very complete and recent survey by Zobel and Moffat [28] for a thorough bibliography on the subject.

Two main complementary techniques are at the basis of index compression: instantaneous codes provide storage for integers that is proportional to the size of the integer (e.g., smaller numbers use less bits); gap encoding turns lists of increasing integers (for instance, the monotonically increasing list of numbers of documents in which a term appears) into lists of small integers, the gaps between successive values (e.g., the difference). The two techniques, combined, make it possible to store inverted indices in highly compressed form. Instantaneous codes are also instrumental in storing in little space information such as the number of documents in which each term appears.

Since inverted indices are so important for search engines, it is not surprising that a large amount of research has studied how to maximize either the speed or the compression ratio of gap-encoded indices. Depending on the application, compression or speed may be considered more important, and different solutions propose different tradeoffs.

In this paper, we describe a new type of compressed index that does not use gaps. Rather, we carefully engineer and tailor to the needs of a search engine a well-known quasi-succinct representation for monotone sequences proposed by Peter Elias [12]. We explain how to code every part of the index by exploiting the bijection between sequence of integers and their prefix sums, and we provide details about the physical storage of our format.

Our new index is theoretically attractive: it guarantees to code the information in the index close to its information-theoretical lower bound, and provides on average constant-time access to any piece of information stored in the index, including searching for elements larger than a given value (a fundamental operation for computing list intersections quickly). This happens by means of a very simple addressing mechanism based on a linear list of forward pointers. Moreover, sequential scanning can be performed using a very small number of logical operation per element. We believe it is particularly attractive for in-memory or memory-mapped indices, in which the cost of disk access is not dominant.

To corroborate our findings, in the last part of the paper, we index the TREC GOV2 collection and a collection of 130 million pages of the .uk web2 with different type of encodings, such as δ and Golomb. We show that, while not able to beat gaps coded with Golomb codes, our index compresses better than γ/δ codes or variable-length byte codes.

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1Incidentally, Elias also invented some of the most efficient codes for gap compression [13].
2We remark that TREC GOV2 is publicly available, and that the latter collection is available from the author.
We then compare a prototype Java implementation of our index against MG4J (using its high-performance indices) and Lucene, two publicly available Java engines, and Zettair, a C search engine. MG4J has been set up to use $\gamma/\delta$ codes, whereas Lucene and Zettair use variable-length byte codes. We get a full confirmation of the good theoretical properties of our index, with excellent timings for conjunctive, phrasal and proximity queries. We also provide some evidence that for pointer lists our index is competitive with the Kamikaze implementation of PForDelta codes [29].

The quasi-succinct indices described in this paper are the default indices used by MG4J from version 5.0. All code used in this paper is available for download under the Lesser GNU GPL.  

2. RELATED WORK

The basis of the current compression techniques for inverted indices is gap encoding, developed at the start of the ‘90s [3]. Gap encoding made it possible to store a positional inverted index in space often smaller than the compressed document collection. Gaps (differences between contiguous document pointers in the posting list) have to be encoded using instantaneous codes that use shorter code-words for smaller integers, and previous research in information theory provided $\gamma$, $\delta$ [13] and Golomb [15] codes, which achieve excellent compression. Moreover, a wealth of alternative codes have been developed in the last 30 years.  

When speed is important, however, such codes are rather slow to decode: in practice, often implementation use the folklore variable-length byte code (e.g., the open-source search engine Lucene, as well as Zettair). Recent research has developed a number of word-aligned codes (e.g., [1]) that encode in a single machine word several integers, providing high-speed decoding and good compression. In [29], the author tailor their PForDelta code to the behavior of modern super-scalar CPUs and their caches.

More specialized techniques tackle specific problems, studying in great detail the behaviour of each part of the index: for instance, [27] studies in great detail the compression of positional information.

Another line of research studies the renumberings of the documents that generate smaller gaps. This phenomenon is known as clustering [20], and can be induced by choosing a suitable numbering for the documents [4, 25, 5].

As indices became larger, a form of self-indexing [21] became necessary to compute quickly the intersection of lists of documents, an operation that is at the basis of the computation of conjunctive Boolean queries, proximity queries and phrasal queries.

The techniques used in this paper are based on a seminal paper by Elias [12], which is a precursor of succinct data structures for indexed sets [23]. We do use some of the knowledge developed by the algorithmic community working on succinct data structures, albeit in practice the theoretical encodings developed there, which concentrate on attaining asymptotically optimal speed using $o(n)$ additional bits, where $n$ is the optimal size for the data structure, have presently too high constant costs to be competitive in real applications with methods using $O(n)$ additional bits.

We remark that the literature on the subject is actually immense, and impossible to recap in this section. The references above should be considered mostly as pointers. We refer the reader again to [19, 28] for a complete historical overview.

3. DEFINITIONS

In this paper we discuss the indexing problem for a collection of documents. We give definitions from scratch as we will need to discuss formally the index content.

Each document is represented by a number, called document pointer, starting from zero. Each document $d$ has a length $\ell$, and is formed by a sequence of terms $t_0, t_1, \ldots, t_{\ell-1}$. For each document and each term, the count specifies how many times a term appears in the sequence forming the document. The frequency is the number of documents in which a term appears (i.e., the number of documents for which the count is not zero). The occurrence of a term is the number of occurrences of the term in the whole collection, that is, the sum of the counts of the term over all documents.

The posting list for a term is the (monotonically increasing) list of documents where the term appears. With each document we associate also the (nonzero) count of the term in the document, and the (monotonically increasing) list of positions (numbered from zero) at which the term appears in the given document.

The unary code associates with the natural number $n \geq 0$ the codeword $0^n1$. The negated unary code associates with the natural number $n \geq 0$ the codeword $1^n0$.

A bit array of length $n$ is a sequence of bits $b_0, b_1, \ldots, b_{n-1}$. We sometime view such an array as a stream: we assume that there is an implicit pointer, and that I/O operations such as reading unary codes are performed by scanning the array and updating the implicit pointer accordingly.

4. QUASI-SUCCINCT REPRESENTATION OF MONOTONE SEQUENCES

In this section we give a detailed description of the high bits/low bits representation of a monotone sequence proposed by Elias [12] and independently by Fano [14]. We assume to have a monotonically increasing sequence of $n > 0$ natural numbers

$$0 \leq x_0 \leq x_1 \leq \cdots \leq x_{n-2} \leq x_{n-1} \leq u,$$

where $u > 0$ is any upper bound on the last value. The choice $u = x_{n-1}$ is of course possible (and optimal), but storing explicitly $x_{n-1}$ might be costly, and a suitable value for $u$ might be known from external information, as we will see shortly. We will represent such a sequence in two bit arrays as follows:

- the lower $\ell = \max\{ 0, \lceil \log(u/n) \rceil \}$ bits of each $x_i$ are stored explicitly and contiguously in the lower-bits array;
- the upper bits are stored in the upper-bits array as a sequence of unary-coded gaps.

In Figure 1 we show an example. Note that we code the gaps between the values of the upper bits, that is, $\lfloor x_i/2^\ell \rceil - \lfloor x_{i-1}/2^\ell \rceil$ (with the convention $x_{-1} = 0$).

The interesting property of this representation is that it uses at most $2 + \lceil \log(u/n) \rceil$ bits per element: this can be easily seen from

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3http://mg4j.di.unimi.it/
4http://vigna.di.unimi.it/
5Alternative approaches, such as interpolative coding [20], have been proposed to code some part of an index, but they lack the direct-access and skipping features that are necessary for fast query resolution.

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6If $u=0$, the list is entirely made of zeroes, and its content is just defined by $n$.
7Actually, Elias discusses just the case in which $u+1$ and $n+1$ are powers of two, but extending his definitions is an easy exercise.
8We remark that, despite a superficial similarity with the upper/lower splitting of Rice and Golomb codes, the representation above has no connection with neither code, as lower bits contain an explicit value, and not a gap.
The two operations give a bijective correspondence between monotone sequences. We consider the list of prefix sums. Conversely, given a list $a_0, a_1, \ldots, a_n$ of natural numbers we can consider the list of prefix sums $s_k = \sum_{i=0}^{k-1} a_i$ for $0 \leq k \leq n$. The two operations give a bijective correspondence between monotone sequences bounded by $a$ and lists of natural numbers of the same length whose sum is bounded by $u$. Thus, we can represent using the high bits/low bits presentation either monotonically increasing sequences, or generic lists of integers.

The quasi-succinct representation above has a number of useful properties that make it quite advantageous over gap-encoded sequences:

- The distribution of the document gaps is irrelevant: there is no code to choose, because the lower bits are stored explicitly in a fixed-width format, and the representation of the upper bits, being made by $n$ ones and at most $2n$ zeroes, is a perfect candidate for the unary code.
- Compression is guaranteed irrespective of gaps being well distributed (e.g., because of correlation between the content of consecutive document) or not. In particular, renumbering documents in a way that improves retrieval speed (e.g., to ease early termination) will not affect the index size.
- Scanning sequentially the list using a longword buffer requires to perform just a unary read and using few shifts for each element.
- In general, the high bits/low bits representation concentrates the difficulty of searching and skipping on a simple bit array of unary codes containing $n$ ones and at most $2n$ zeroes. We can devise extremely fast, practical ad hoc techniques that exploit this information.

Actually, Elias’s original paper suggests the most obvious solution for quick (on average, constant-time) reading of a sequence of unary codes: we store forward pointers to the positions (inside the upper-bits array) that one would reach after $kq$ unary-code reads, $k \geq 0$, where $q$ is a fixed quantum (in other words, we record the position immediately after the one of index $kq-1$ in the bit array). Retrieving $x_i$ now can be done by simulating $q\lfloor i/q \rfloor$ unary reads using a forward pointer, and completing sequentially with $i \mod q < q$ unary-code reads. On average, by (1), the sequential part will read at most $3q$ bits. Smaller values of $q$ yield less reads and use more space.

**Skipping.** A more interesting property, for our purposes, is that by storing skip pointers to positions reached after negated unary-code reads of the upper bits it is possible to perform skipping, that is, to find very quickly, given a bound $b$, the smallest $x_i \geq b$. This operation is fundamental in search engines as it is the base for quick list intersection.

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9Actually, the representation is one of the ingredients of sophisticated, modern succinct data structures that attain the information-theoretical bound [23].

10Note that sequences of prefix sums contain an additional element $s_0 = 0$ that is not part of the bijection.

11The same bijection is used normally to code monotone sequences using gaps, but we intend to to the opposite.

12Prefix sums have indeed several applications in compression, for instance to the storage of XML documents [10].

13This problem is essentially (i.e., modulo an off-by-one) the selection problem for which much more sophisticated solutions, starting with Clarke’s [8], have in the last years shown that constant-time access can be obtained using $o(n)$ additional bits instead of the $O(n)$ bits proposed by Elias and Fano, but such solutions, while asymptotically optimal, have very high constant costs. Nonetheless, there is a large body of theoretical and practical knowledge that has been accumulated in the last 20 years about selection, and we will use some of the products of that research to read multiple unary codes quickly in the upper-bits array.

14Elias describes a slightly different analogous operation, by which he finds the largest $x_i \leq b$; the operation involves moving backwards in the bit array, something that we prefer to avoid for efficiency. Note that this is again essentially equivalent to predecessor search, a basic problem in fast retrieval on sets of integers for which very strong theoretical results are known in the RAM model [22].
To see why this is possible, note that by definition in the upper-bits array the unary code corresponding to the smallest $x_j \geq b$ must terminate after $\lceil b/2^k \rceil$ ones. We could thus perform $\lceil b/2^k \rceil$ negated unary-code reads, getting to position $p$, and knowing that there are exactly $p - \lceil b/2^k \rceil$ ones and $\lceil b/2^k \rceil$ zeroes to our left (i.e., we are in the middle of the unary code for $x_{p-\lceil b/2^k \rceil}$). From here, we complete the search exhaustively, that is, we actually compute the values of the elements of the list (by reading unary codes and retrieving the suitable lower bits) and compare them with $b$, as clearly the element we are searching for cannot be represented earlier in the upper-bits array. An example is shown in Figure 2.

By setting up an array of skip pointers analogously to the previous case (i.e., forward pointers), the reading of negated unary-codes can be perform quickly. Note, however, that in general without further assumptions it is not possible to bound the number of bits read during the $\lfloor b/2^k \rfloor \mod q$ negated unary-code reads that must be performed after following a skip pointer, as there could be few zeroes (actually, even none) in the bit array. Nonetheless, if a linear lower bound on the number of zeroes in the bit array is known, it can be used to show that skipping is performed in constant time on average.

Strictly monotone sequences. In case the sequence $x_0, x_1, \ldots, x_{n-1}$ to be represented is strictly monotone (or, equivalently, the $a_i$’s are nonzero), it is possible to reduce the space usage by storing the sequence $x_i - i$ using the upper bound $u - n$. Retrieval happens in the same way—one just has to adjust the retrieved value for the $i$-th element by adding $i$. This mechanism was already noted by Elias [11] (more generally for $k$-spaced sequences, $k > 0$), but it is important to remark that under this representation the algorithm for skipping will no longer work. This happens because $x_i$ is actually represented as $x_i - i$, so skipping $\lceil b/2^k \rceil$ negated unary codes could move us arbitrarily after the element we would like to reach.

5. SEQUENCES AS A RANKED CHARACTERISTIC FUNCTIONS

In some cases, the quasi-succinct representation we described is not very efficient in term of space: this happens, for instance, for very dense sequences. There is however an alternate representation for strictly monotone sequences with skipping: we simply store a list of $u$ bits in which bit $k$ is set if $k$ is part of the list $x_0, x_1, \ldots, x_{n-1}$. This is equivalent to storing the list in gap-compressed form by writing in unary the gaps $x_i - x_{i-1} - 1$, and guarantees by definition that no more than $u$ bits will be used.

Skipping in such a representation is actually trivial: given the bound $b$, we read a unary code starting at position $b$. The new position $x_i$ is such that $x_i$ is the smallest element satisfying $x_i \geq b$. The only problem is that at this point we will have lost track of the index $i$.

To solve this problem, we take a dual approach to that of the previous section and store a simple ranking structure: for each position $k q$, where $q$ is the quantum, we store the number of ones to the left. After a skip, we simply rank the current position $x_i$ by first reading the precomputed number of ones before $\lfloor x_i/q \rfloor$, and then computing the number of ones in the at most $q$ remaining bits.

6. REPRESENTING AN INVERTED INDEX

We now discuss how the quasi-succinct representation presented in the previous section can be used to represent the posting list of a term. We defer to the next section a detailed discussion of the data-storage format.

Pointers. Document pointers form a strictly monotone increasing sequence. We store them using the standard representation (i.e., not the specialized version for strictly monotone sequences), so to be able to store skip pointers, as skipping is a frequent and useful operation (e.g., during the resolution of conjunctive Boolean queries or phrasal queries), whereas random access to document pointers is not in general necessary. The upper bound is the number of documents $N$ minus one, and the number of elements of the list is $f$, the frequency.

We remark that the apparent loss of compression due to the necessity of using the standard representation (to make skipping possible) turns actually into an advantage: if the last pointer in the list is equal to $a N$, with $0 \leq a < 1$, since $N \geq f$, we can write $N = df + r$ with $d > 0$ and $0 \leq r < f$, and then we have

$$\frac{aN}{2^k} = \frac{a(df + r)}{2 \log(df + r)/f} \geq \frac{a(df + r)}{2 \log d} \geq af. \quad (2)$$

In other words, the slight redundancy guarantees that there are at least $af$ zeroes in the upper-bits array: if $a \approx 1$, we can thus guarantee that on average skipping can be performed in a constant number of steps, as, on average, reading a one implies reading at least a zero, too (and viceversa). Since we write forward pointers only for lists with $f \geq q$, under realistic assumptions on $q$ in practice $a$ is close to 1.

Finally, even in pathological cases (i.e., a every uneven distribution of the zeroes in the list), one every $2^k \leq N/f$ bits must necessarily be zero, as the list is strictly monotone. Thus, terms with dense posting lists must have frequent zeroes independently of the considerations above.

Note that if

$$f + \left\lfloor N/2^k \right\rfloor + f \ell > N$$

then the representation above uses more than $N$ bits (in practice, this happens when $f \gtrsim N/3$). In this case, we switch to a ranked characteristic function. Since there are at most two zeroes for each one in the bitmap, it is easy to check that all operations can still be performed in average constant time.

Counts. Counts are strictly positive numbers, and can be stored using the representation for strictly monotone sequences to increase compression. In this case the upper bound is the occurrence of the term, and the number of elements is again the frequency.

Positions. The format for positions is the trickiest one. Consider, for the $i$-th document pointer in the inverted list for term $t$ with count $c_t$, the list of positions $p_0^t, p_1^t, \ldots, p_{c_t-1}^t$. First, we turn this list into a list of strictly positive smaller integers:

$$p_0^t + 1, p_1^t - p_0^t, p_2^t - p_1^t, \ldots, p_{c_t-1}^t - p_{c_t-2}^t.$$  

Consider the concatenation of all sequences above:

$$p_0^t + 1, p_1^t - p_0^t, p_0^t, \ldots, p_{c_t-1}^t - p_{c_t-2}^t,\ldots,$$

and store them using the representation for strictly positive numbers. In this case it is easy to check that the best upper bound is

$$f + \sum_{0 \leq i < f} p_{c_t-1}^t,$$

and the number of elements is the occurrence $g$ of the term.

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15Nothing prevents from storing both kind of pointers. The increase in size of the index would be unnoticeable.
Figure 2: An example of skipping based on the sequence shown in Figure 1. On the left we have the upper-bits array, and on the right the lower-bits array. We want to skip to the first item larger than or equal to 22, so since \( \ell = 2 \) we have to perform \( 22/2^4 = 5 \) negated unary-code reads (the continuous arrows), getting to position 9, so we are positioned in the middle of the unary code associated with the element of index \( 9 - 5 = 4 \). Then we perform a unary-code read (the dashed arrow), which returns 3, so we know that the upper bits of the current element (of index 4) are \( 3 + 5 = 8 \). Since the block of lower bits of index 4 is zero, we return 32. If we had at our disposal a skip pointer for \( q = 4 \) (the dotted arrow), we could have skipped the first four negated unary-code reads. Note that in general more than one unary-code read might be necessary after reading the negated unary codes.

We now show how to retrieve the positions of the \( i \)-th document. Let \( s_0, s_1, \ldots, s_f \) be prefix sums of the counts (e.g., \( c_i = s_i+1 - s_i \)). We note that the list provides the starting and ending point of the sequence of positions associated to a document: the positions of document \( i \) occur in (3) at positions \( j \) satisfying \( s_i \leq j < s_i+1 \). Let \( t_0, t_1, \ldots, t_e \) be the sequence of prefix sums of the sequence (3). It is easy to check that the positions of \( i \)-th document can be recovered as follows:

\[
p'_j = t_{s_i+j+1} - t_{s_i} - 1 \quad 0 \leq j < c_i.
\]

We remark that the nice interplay between prefix sums and lists of natural numbers is essential in making this machinery work: we need the counts \( c_i \) (e.g., to compute a content-based ranking function), but we need also their prefix sums to locate positions.

7. A QUASI-SUCCINCT BITSTREAM

We now discuss in detail the bit stream used to store the quasi-succinct representation described in Section 4—in particular, the sizing of all data involved.

Metadata pertaining the whole representation, if present, can be stored initially in a self-delimiting format. Then, the remaining data is laid out as follows: pointers, lower bits, upper bits (see Figure 3). The rationale behind this layout is that the upper-bits array is the only part whose length is in principle unknown: by positioning it at the end of the bitstream, we do not have to store pointers to the various parts of the stream. The lower-bits array will be located at position \( sw \), where \( s \) is the number of pointers and \( w \) their width, and the upper-bits array at position \( pw + nw \) bits after the metadata.

We can thus compute without further information the starting point of each element of the stream.

We assume that the number of elements \( n \) is known, possibly from the metadata. The first issue is thus the size and the number of pointers. If the upper bound \( u \) is known, we know that the upper-bits array is \( n + \lceil u/2^k \rceil \) bits long at most, so the width of the pointers is \( w = \lceil \log(n + \lceil u/2^k \rceil + 1) \rceil \); otherwise, information must be stored in the metadata part so to be able to compute \( w \).

If we are storing forward pointers for unary codes, the number of pointers will be exactly \( \lceil n/q \rceil \); otherwise (i.e., if we are storing forward pointers for negated unary codes), they will be at most \( s = \lfloor (n + \lceil u/2^k \rceil)/q \rfloor \).\(^{16}\) Again, if the bound \( u \) is not known it is necessary to store information in the metadata part so to be able to compute \( s \).

Analogously, if \( u \) is not known we need to store metadata that makes us able to compute \( \ell = \lceil \log(u/n) \rceil \).

Finally, in the case of a ranked characteristic functions instead of pointers we store \( \lfloor f/q \rfloor \) cumulative ranks of width \( w = \lceil \log N \rceil \), followed by the bitmap representation of the characteristic function.

8. LAYING OUT THE INDEX STRUCTURE

We now show how to store in a compact format all metadata that are necessary to access the lists. For each index component (document pointers, counts, positions) we write a separate bit stream. We remark that for an index that provides naturally constant-time access to each element, there is no point in interleaving data, and this is another advantage of quasi-succinct encoding, as unnecessary data (e.g., counts and positions for a Boolean query) need not be examined. As usual, for each term we store three pointers locating the starting point of the information related to that term in each stream.

The bit stream for document pointers contains as metadata the frequency and the occurrence of the term. We write the occurrence in \( \gamma \) code and, if the occurrence is greater than one, the difference between occurrence and frequency, again in \( \gamma \) code (this ensures that hapaxes use exactly one bit). This information, together with the number of documents in the collection, is sufficient to access the quasi-succinct representation of document pointers (see Section 6).

The bit stream for counts contains no metadata. The occurrence and frequency can be obtained from the pointers stream, and they are sufficient to access the representation.

The bit stream for positions requires to store in the metadata part the parameter \( \ell \) and the skip-pointer size \( w \), which we write again in \( \gamma \) code, as the upper bound (4) is not available. Note that if the occurrence is smaller than \( q \), there is no pointer, and in that case we omit the pointer size. Thus, the overhead for terms with a small number of occurrences is limited to the parameter \( \ell \).\(^{17}\)

9. IMPLEMENTATION DETAILS

Implementation details are essential in a performance-critical data structure such as an inverted index. In this section we discuss the main ideas used in our implementation. While relatively simple, these ideas are essential in obtaining, besides good compression, a significant performance increase.

Longword addressing. We either load the index into memory, or access it as a memory-mapped region. Access happens always by longword, and shifts are used to extract the relevant data. The bit \( k \) of the index is represented in longword \( \lfloor k/64 \rfloor \) in position \( k \) mod 64. While direct access to every point of the bitstream is possible, we keep track of the current position so that sequential reads use the

\(^{16}\)We remark that if \( u > n(q-1) \) some of the \( q \) pointers might actually be unused. It is sufficient to set them to zero (no other pointer can be zero) and consider them as skips to the end of the list.

\(^{17}\)Actually, it is easy to check that the overhead for hapaxes is exactly 2 bits with respect to writing the only existing position in \( \delta \) code.
last longword read as a bit buffer. Extraction of lower bits requires very few logical operations in most cases when \( \ell \) is small.

**Reading unary codes.** Reading a unary code is equivalent to the computation of the least significant bit. We use the beautiful algorithm based on de Bruijn’s sequences [18], which is able to locate the least significant bit using a single multiplication and a table lookup. The lack of any test makes it a very good choice on superscalar processors, as it makes prediction and out-of-order execution possible.\(^\text{18}\)

Both when looking up an entry and when skipping, we have, however, to perform a significant number of unary-code reads (on average, \( \approx q/2 \)). To this purpose, we resort to a *broadword* (a.k.a. SWAR, i.e., “SIMD in A Register”) *bit search* [26]. The idea is that of computing the number of ones in the current bit buffer using the classical algorithm for *sideways addition* [17], which involves few logical operations and a multiplication. If the number of reads we have to perform exceeds the number of ones in the current buffer, we examine the next longword, and so on. Once we locate the right longword, we can complete the search using the broadword *selection* algorithm presented in [26].

Our experiments show that broadword bit search is extremely effective, unless the number of reads is very small, as in that case computing iteratively the least significant bit becomes competitive. Indeed, when skipping a very small number of position (e.g., less than eight) we simply resort to iterating through the list.

**Cache the last prefix sum.** When retrieving a count or the first position of a position list, we have, in theory, to compute two associated prefix sums. During sequential scans, however, we can cache the last computed value and use it at the next call. Thus, in practice, scanning sequentially counts or positions requires just one unary-code read and one fixed-width bit extraction per item. Reading counts is however made slower by the necessity to compute the difference between the current and the previous prefix sum.

**Trust the processor cache.** The cost of accessing an in-memory index is largely dominated by cache misses. It is thus not surprising that using a direct access (i.e., by pointer) can be slower than actually scanning linearly the upper-bits array using a broadword bit search if our current position is close to the position to get to. The threshold depend on architectural issues and must be set experimentally. In our code we use \( q = 256 \) and we do not use pointers if we can skip to the desired position in less that \( q \) reads.\(^\text{19}\) An analogous strategy is used with ranked characteristic functions: if we have to skip in the vicinity of the current position and the current index is known we simply read the bitmap, using the sideways addition algorithm to keep track of the current index.

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18 Actually, we first check whether we can compute the least significant bit using an 8-bit precomputed table, as the guaranteed high density of the upper bits makes this approach very efficient.

19 Remember, again, that we will actually simulate such reads using a broadword bit search.

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10. **EXPERIMENTS**

We have implemented the quasi-succinct index described in the previous section in Java, and for the part related to document pointers and count, in C++. All the code used for experiments is available at the MG4J web site. In this section, we report some experiments that compare its performance against three competitors:

- Lucene, a very popular open-source Java search engine (release 3.6.0);
- the classical high-performance indices from MG4J [6], another open-source search engine (release 5.0);
- Zettair, a search engine written in C by the Search Engine Group at RMIT University.
- The Kamikaze\(^\text{20}\) library, implementing the PForDelta [29] sequence compression algorithm (up-to-date repository version from GitHub);
- We compare also with recent optimized C code implementing PForDelta compression document pointers and count kindly provided by Ding Shuai [24].

Zettair has been suggested by the TREC organizers as one of the baselines for the efficiency track. The comparison of a Java engine with a C or C++ engine is somewhat unfair, but we will see actually the Java engines turn out to be always significantly faster.

We use several datasets summarized in Table 1: first, the classical public TREC GOV2 dataset (about 25 million documents) and a crawl of around 130 million pages from the .uk domain that is available from the author. Tokens were defined by transition between alphanumerical to nonalphanumerical characters or by HTML flow-breaking tags, and they were stemmed using the Porter2 stemmer.\(^\text{21}\) Besides an index considering the whole HTML document, we created some indices for the title text only (e.g., the content of the HTML TITLE element), as such indices have significantly different statistics (e.g., documents are very short).

Additionally, we created a *part-of-speech* index used within the Mimir semantic engine [9]; such indices have a very small number of terms that represent syntactic elements (nouns, verbs, etc.), very dense posting lists and a large number of positions per posting: they provide useful information about the effectiveness of compression when the structure of the index is not that of a typical web text index. For the same reason, we also index a collection of about a dozen millions tweets from Twitter.

Small differences in indexing between different search engines are hard to track: the details of segmentation, HTML parsing, and so on, might introduce discrepancies. Thus, we performed all our

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20 \url{http://sna-projects.com/kamikaze/}

21 Zettair, however, supports apparently only the original Porter stemmer.
indexing starting from a pre-parsed stream of UTF-8 text documents. We also checked that the frequency of the terms we use in our queries is the same—a sanity check showing that the indexing process is consistent across the engines. Finally, we checked that the number of results of conjunctive and phrasal queries was consistent across the different engines, and that bpref scores were in line with those reported by participants to the Terabyte Track.

Using MG4J, we have created indices that use γ codes for counts, and either δ or Golomb codes for pointers and positions22, endowed with a mild amount of skipping information using around 1% of the index size: we chose this value because the same amount of space is used by our index to store forward and skip pointers when \( q = 256 \). These indices (in particular, the ones based on Golomb codes) are useful to compare compression ratios: if speed is not a concern, they provide very good compression, and thus they provide a useful reference points on the compression/speed curve.23

We remark that we have indexed every word of the collections. No stopword elimination has been applied. Commercial search engines (e.g., Google) are effortlessly able to search for the phrase “Romeo and Juliet”, so our engine should be able to do the same.

**Compression.** Table 2 reports a comparison of the compression ratios. Our quasi-succinct index compresses always better than γ/δ, but worse than Golomb codes. In practice, our index reduces the size of the γ/δ index by \( \approx 10\% \), whereas Golomb codes reach \( \approx 20\% \).

The compression of Lucene and Zettair on the text of web pages is not very good (a \( \approx 15\% \) increase w.r.t. our index). This was partially to be expected, as both Lucene and Zettair use variable-length byte codes for efficiency, and while such codes are easy to decode, they are ill-suited to compression. When the distribution of terms and positions is different, however, compression is significantly worse: for titles we have a 50% increase in size, and for the Mímir semantic index or tweets a 40% increase. This is somewhat typical: variable-length byte codes compress most positions in a single byte if the distribution of words comes from a “natural” distribution on documents of a few thousand words. Using shorter documents (e.g., titles and tweets) or a different distribution (e.g.,

<table>
<thead>
<tr>
<th>Documents</th>
<th>Terms</th>
<th>Postings</th>
<th>Occurrences</th>
</tr>
</thead>
<tbody>
<tr>
<td>TREC GOV2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Text</td>
<td>25 M</td>
<td>35 M</td>
<td>5.5 G</td>
</tr>
<tr>
<td>Title</td>
<td>25 M</td>
<td>1.1 M</td>
<td>135 M</td>
</tr>
<tr>
<td>Web .uk</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Text</td>
<td>132 M</td>
<td>99 M</td>
<td>21 G</td>
</tr>
<tr>
<td>Title</td>
<td>132 M</td>
<td>3.2 M</td>
<td>609 M</td>
</tr>
<tr>
<td>Mímir index</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tokens</td>
<td>1 M</td>
<td>49</td>
<td>27 M</td>
</tr>
<tr>
<td>Tweets</td>
<td>13 M</td>
<td>2.3 M</td>
<td>147 M</td>
</tr>
</tbody>
</table>

Table 1: Basic statistics for the datasets used in our experiments.

<table>
<thead>
<tr>
<th>QS</th>
<th>MG4J γ/δ</th>
<th>Golomb</th>
<th>Lucene</th>
<th>Zettair</th>
</tr>
</thead>
<tbody>
<tr>
<td>TREC GOV2 (text)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pointers</td>
<td>7.59</td>
<td>8.47</td>
<td>6.94</td>
<td></td>
</tr>
<tr>
<td>Counts</td>
<td>2.98</td>
<td>2.56</td>
<td>—</td>
<td></td>
</tr>
<tr>
<td>Positions</td>
<td>10.17</td>
<td>11.11</td>
<td>8.65</td>
<td></td>
</tr>
<tr>
<td>Overall</td>
<td>37.0 GB</td>
<td>40.3 GB</td>
<td>31.9 GB</td>
<td>42.1 GB</td>
</tr>
<tr>
<td>TREC GOV2 (title)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pointers</td>
<td>10.29</td>
<td>11.44</td>
<td>9.54</td>
<td></td>
</tr>
<tr>
<td>Counts</td>
<td>1.10</td>
<td>1.14</td>
<td>—</td>
<td></td>
</tr>
<tr>
<td>Positions</td>
<td>3.84</td>
<td>4.63</td>
<td>3.05</td>
<td></td>
</tr>
<tr>
<td>Overall</td>
<td>268 MB</td>
<td>308 MB</td>
<td>241 MB</td>
<td>396 MB</td>
</tr>
<tr>
<td>Web .uk (text)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pointers</td>
<td>8.59</td>
<td>9.72</td>
<td>7.98</td>
<td></td>
</tr>
<tr>
<td>Counts</td>
<td>2.39</td>
<td>2.06</td>
<td>—</td>
<td></td>
</tr>
<tr>
<td>Positions</td>
<td>10.16</td>
<td>10.95</td>
<td>8.41</td>
<td></td>
</tr>
<tr>
<td>Overall</td>
<td>108 GB</td>
<td>117 GB</td>
<td>92 GB</td>
<td>126 GB</td>
</tr>
<tr>
<td>Web .uk (title)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pointers</td>
<td>11.92</td>
<td>13.51</td>
<td>11.27</td>
<td></td>
</tr>
<tr>
<td>Counts</td>
<td>1.13</td>
<td>1.18</td>
<td>—</td>
<td></td>
</tr>
<tr>
<td>Positions</td>
<td>4.36</td>
<td>5.06</td>
<td>3.35</td>
<td></td>
</tr>
<tr>
<td>Overall</td>
<td>1.39 GB</td>
<td>1.59 GB</td>
<td>1.26 GB</td>
<td>2.00 GB</td>
</tr>
<tr>
<td>Mímir token index</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pointers</td>
<td>1.51</td>
<td>1.42</td>
<td>1.48</td>
<td></td>
</tr>
<tr>
<td>Counts</td>
<td>6.42</td>
<td>6.28</td>
<td>—</td>
<td></td>
</tr>
<tr>
<td>Positions</td>
<td>5.83</td>
<td>6.22</td>
<td>5.03</td>
<td></td>
</tr>
<tr>
<td>Overall</td>
<td>0.96 GB</td>
<td>1.01 GB</td>
<td>0.83 GB</td>
<td>1.34 GB</td>
</tr>
<tr>
<td>Tweets</td>
<td>10.13</td>
<td>10.29</td>
<td>9.22</td>
<td></td>
</tr>
<tr>
<td>Counts</td>
<td>1.06</td>
<td>1.11</td>
<td>—</td>
<td></td>
</tr>
<tr>
<td>Positions</td>
<td>4.67</td>
<td>5.94</td>
<td>3.86</td>
<td></td>
</tr>
<tr>
<td>Overall</td>
<td>302 MB</td>
<td>341 MB</td>
<td>266 MB</td>
<td>423 MB</td>
</tr>
</tbody>
</table>

Table 2: A comparison of index sizes. We show the overall index size, which includes skipping structures, and, if available, the number of bits per element of each component, excluding skipping structures.

---

22The Golomb modulus has been chosen separately for each document. The results we obtain seems to be within 5% of the best compression results obtained in [27], which suggest a space usage of 21 MB/query on average for an average of 20.72 millions positions per query. A more precise estimate is impossible, as results in [27] are based on 1000 unknown queries, and no results about the whole GOV2 collection are provided.

23We have also tried interpolative coding [20], but on our collections the difference in compression with Golomb codes was really marginal.
a semantic index) yields very bad results. A 50% increase in size, indeed, can make a difference.

While we are not aiming at the best possible compression, but rather at high speed, it is anyway relieving to know that we are improving (as we shall see shortly) both compression and speed with respect to these engines.

Interestingly, counts are the only index component for which we obtain sometimes worse results than \( \gamma \) coding. This is somewhat to be expected, as we are actually storing their prefix sums. The impact of counts on the overall index, however, is quite minor, as shown by the small final index size.

**Speed.** Benchmarking a search engine brings up several complex issues. In general, the final answer is bound to the architecture on which the tests were run, and on the type of queries. A definite answer can be given only against a real workload. Our tests were performed on a recent workstation sporting a 3.4 GHz Intel i7-3770 CPU with 8 MiB of cache and 16 GiB of RAM.

We aim at comparing speed of in-memory indices, as one of the main reasons to obtain smaller indices is to make more information fit into memory; moreover, the diffusion of solid-state disks makes this approach reasonable. Thus, in our tests we resolve each query three times before taking measurements. In this way we guarantee that the relevant parts of the index have actually been read and memory mapped (for MG4J and Lucene, or at least cached by the file system, for Zettair), and we also make sure that the Java virtual machine is warmed up and has performed inlining and other runtime optimizations. With this setup, our tests are highly repeatable and indeed the relative standard deviation over several runs is less than 3%.

We used the 150 TREC Terabyte track (2004–2006) title queries in conjunctive, phrasal and proximity form (in the latter case, the terms in the query must appear in some order within a window of 16 words). We also extracted the terms appearing in the queries and used them as queries to measure pure scanning speed: all in all, we generated 860 queries. MG4J and Lucene were set up to compute the query results without applying any ranking function. Zettair was set to Okapi BM25 ranking [16], which appeared to have the smaller impact on the query resolution time (no “no-ranking” mode is available).

All engines were set up to return a single result, so that the logic needed to keep track of a large result size would not interfere with the evaluation. The results are shown in Table 3. The first column (QS) shows the results of query resolution on a quasi-succinct index. The third column (MG4J) for a \( \gamma \)-coded high-performance MG4J index. The fourth column for Lucene, and the last column for Zettair.

The second column (QS*) needs some explanation. Both Lucene and MG4J interleave document pointers and counts. As a consequence, resolving a pure Boolean query has a higher cost (as counts are read even if they are not necessary), but ranked queries require less memory/disk access. To simulate a similar behaviour in our setting, we modified our code so to force it to read the count of every returned document pointer. This setting is of course artificial, but it provides a good indication of the costs of iterating and applying a count-based ranking function, and it will be the basis of our comparison. For phrasal and proximity queries there is no difference between QS and QS* as counts have in any case to be read to access positions.

First of all, we note that decoding a quasi-succinct index is slightly (\( \approx 7\% \)) faster than decoding a gap-compressed index that uses variable-byte codes. It is nonetheless important to notice that our timings for purely boolean resolution (QS) are much lower, and this can be significant in a complex query (e.g., a conjunction of disjunctively expanded terms). Zettair is much slower.

More interestingly, we have a \( \approx 50\% \) improvement for conjunctive queries, a \( \approx 40\% \) improvement for phrasal queries and a \( \approx 60\% \) improvement for proximity queries: being able to address in average constant time every element of the index is a real advantage. We also remind the reader that we are comparing a Java prototype with a mature implementation.

We expect the asymptotic advantage of quasi-succinct indices to be more evident as the collection size grows. To test this hypothesis, we performed further experiments using the Web .uk collection and 1000 multi-term queries randomly selected from a large search-engine query log. The results are shown in Table 6: now conjunctive and proximity queries are more thrice faster with respect to Lucene.

In Table 4 we show some data comparing in-memory quasi-succinct indices with PForDelta code. The data we display is constrained by some limitations: the Kamikaze library does not provide count storage; and the optimized C code we are using [24] does not provide positions. This is an important detail, as quasi-succinct indices trade some additional efforts in decoding counts (i.e., computing their prefix sums) in exchange for constant-time access to positions. Our main goal is to speed up positional access—indeed, nothing prevents using PForDelta for document pointers and storing counts and positions as described in this paper (or even using a separate PForDelta index without positions as a first-pass index).

Kamikaze turns out to be slightly slower for scanning term lists, and almost twice as slow when computing conjunctive queries. To estimate the difference in compression, we computed the space used by the document pointers of our TREC collections using Kamikaze: the result is an increase of \( \approx 55\% \) in space usage. While not extremely relevant for the index size (positions are responsible mostly for the size of an index), it shows that we would gain no advantage from storing pointers using PForDelta in a Java engine.25

The comparison of C implementations, on the other hand, is definitely in favour of PForDelta: apart from pointer enumeration our C implementation is slower, in particular when enumerating terms and their counts.

There are some important caveats, however: the code we have been provided for PForDelta testing [24] is a bare-bone, heavily optimised C benchmarking implementation that is able to handle only 32-bit document pointers and has a number of limitations such as hardwired constants (e.g., the code needs to be recompiled if the number of document in the collection changes). Our C++ code is a 64-bit fully usable implementation derived from a line-by-line translation of our Java prototype code that could be certainly improved by applying CPU-conscious optimizations. A more realistic comparison would require a real search engine using PForDelta to solve queries requiring positional information, it happens in Table 3.26 Another important aspect that makes the comparison difficult is that quasi-succinct indices compute, beside the count of each document, the cumulative count, which is then used to access

24Note that in real-world search engines the queries that are actually solved are very different by those input by the user, as they undergo a number of rewritings. As a consequence, blindly analyzing queries from large query logs in disjunctive or conjunctive mode cannot give a reliable estimate the actual performance of an index.

25Note that storing positions with PForDelta codes is known to give a compression rate close to that provided by variable-byte coding [27].

26Such an engine is not available, to the best of the authors’s knowledge. The authors of [27] have refused to make their engine available for replication of their results.
quickly positions. In practise, part of the work that is necessary to access positions is moved into the count computation. While this makes the count access slower, it provides exceptional performance when positions are accessed, as our benchmarks show.

Finally, quasi-succinct indices improve by definition their relative performance as the collection gets larger: in Table 6 we report similar data for our Web .uk collection, and conjunctive queries are only \( \approx 13\% \) slower than PForDelta, instead of \( \approx 21\% \).

11. CONCLUSIONS

We have presented a new inverted index based on the quasi-succinct encoding of monotone sequences introduced by Elias and Fano, and on ranked characteristic functions. The new index provides better compression than typical gap-encoded indices, with the exception of extremely compression-oriented techniques such as Golomb or interpolative coding. When compared with indices based on gap compression using variable-length byte encoding (Lucene) or \( \gamma/\delta \) codes (MG4J), not only do we provide better compression, but significant speed improvement over conjunctive, phrasal and proximity queries. In general, any search engine accessing positional information for selecting or ranking documents out of a large collection would benefit from quasi-succinct indices (an example being tagged text stored in parallel indices).

Our comparison with a C implementation of PForDelta compression for pointers and counts showed that PForDelta is slightly faster than quasi-succinct indices in computing conjunction, and significantly faster at retrieving counts, albeit in queries mixing terms with high and low frequency (benchmarks not shown here) quasi-succinct indices can be extremely faster. As the collection size grows, moreover, quasi-succinct indices becomes relatively faster.

For what matters size, PForDelta (more precisely, the Kamikaze library) use \( 55\% \) more space than a quasi-succinct index to compress pointers from the GOV2 collection.

A drawback of quasi-succinct indices is that some basic statistics (in particular, frequency, occurrence and the bound (4)) must be known before the index is built. This implies that to create a quasi-succinct index from scratch it is necessary to temporary cache in turn each posting list (e.g., using a traditional gap-compressed format) and convert it to the actual encoding only when all postings have been generated. While it is easy to do such a caching offline, it could slow down index construction.

On the other hand, this is not a serious problem: in practice, large indices are built by scanning incrementally (possibly in parallel) a collection, and merges are performed periodically over the resulting segments (also called barrels or batches). Since during the construction of a segment it is trivial to store the pieces of information that are needed to build a quasi-succinct index, there is no need for an actual two-pass construction: segments can be compressed using gap encoding, whereas large indices can be built by merging in a quasi-succinct format.

Note that if computing selection-in-a-word and sideways addition were available in hardware (the latter was actually available in the SPARC architecture), the decoding speed of a quasi-succinct index would significantly increase. Moreover, faster hardware selection of the least significant bit would increase significantly the decoding speed. In Java virtual machines, this would lead a to better intrinsification of the method Long.numberOfTrailingZeros(), whereas the gcc compiler could provide faster versions of built-in functions such as \texttt{__builtin_ctzll()}.

All in all, we believe that quasi-succinct indices occupy a very interesting and previously unoccupied spot in the size/speed trade-off curve, in particular for documents (e.g., tweets) that are different in distribution than web pages, and for queries that involve positional access. We remark that there is an obvious advantage in speed in having an index that is significantly smaller, as a larger part of the index can be cached.

An interesting area of future research would be extending the techniques described in this paper to impact-sorted indices, in which documents are sorted following a retrieval-based impact order [2], and only documents pointers with the same impact are monotoni- cally increasing. A technique similar to that used in this paper to store positions (i.e., a different encoding for the start of each block) might provide new interesting tradeoffs between compression and efficiency.

12. ACKNOWLEDGMENTS

The author would like to thank Roi Blanco for an uncountable number of useful suggestions and for moral support.

13. REFERENCES

Table 4: Timings in seconds for running the term and conjunctive test queries from the TREC Terabyte track on GOV2 directly from RAM. Timings for quasi-succinct indices are provided both for Java and C++ 64-bit implementations. PForDelta timings have been computed both using the Kamikaze library and using optimized 32-bit C code provided by Ding Shuai [24]. Starred versions include reading counts for all returned document pointers.

<table>
<thead>
<tr>
<th></th>
<th>QS</th>
<th>QS*</th>
<th>Kamikaze</th>
<th>QS(C)</th>
<th>QS*(C)</th>
<th>PFD(C)</th>
<th>PFD*(C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Terms</td>
<td>3.83</td>
<td>7.30</td>
<td>4.23</td>
<td>1.61</td>
<td>4.05</td>
<td>1.57</td>
<td>2.39</td>
</tr>
<tr>
<td>And</td>
<td>1.16</td>
<td>1.62</td>
<td>2.08</td>
<td>0.91</td>
<td>1.25</td>
<td>0.75</td>
<td>0.87</td>
</tr>
</tbody>
</table>

Table 5: Timings in seconds for running 1000 randomly selected queries from a search-engine query log on the Web .uk collection. See also Table 3.

<table>
<thead>
<tr>
<th></th>
<th>QS(C)</th>
<th>QS*(C)</th>
<th>PFD(C)</th>
<th>PFD*(C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Terms</td>
<td>23.8</td>
<td>56.8</td>
<td>23.6</td>
<td>31.6</td>
</tr>
<tr>
<td>And</td>
<td>19.2</td>
<td>24.5</td>
<td>16.9</td>
<td>19.4</td>
</tr>
</tbody>
</table>

Table 6: Timings in seconds for running 1000 randomly selected queries from a search-engine query log on the Web .uk collection. See also Table 4.


