In-core computation of distance distributions and geometric centralities with HyperBall: A hundred billion nodes and beyond

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- You want to understand which nodes are *important* in some sense

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By JOHN MARKOFF and SOMINI SENGUPTA

Published: November 21, 2011

The world is even smaller than you thought.

RECOMMEND

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Geometric Centralities

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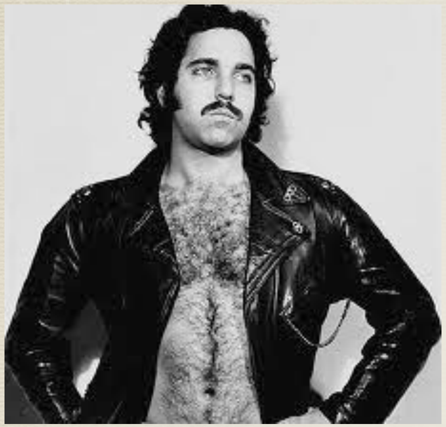
- The (preliminary) results show that harmonic centrality has a very good signal (in fact, better NDCG@10/P@10 than anything we tried)
- In general, HyperBall makes it possible to use harmonic centrality on very large graphs

Hollywood: PageRank

Ron Jeremy



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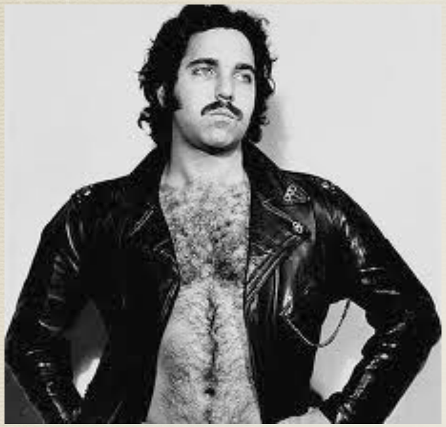

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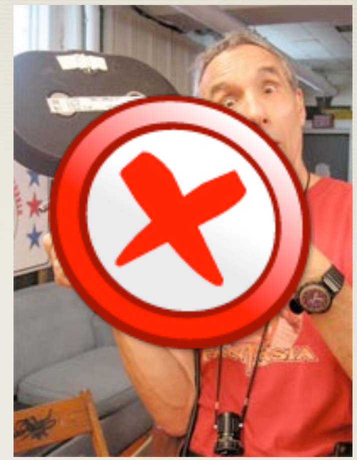

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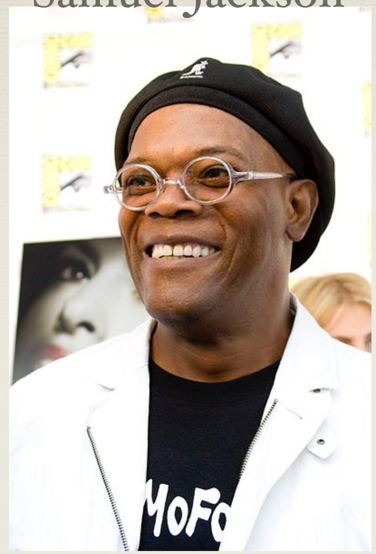
Hollywood: **Harmonic**

George Clooney

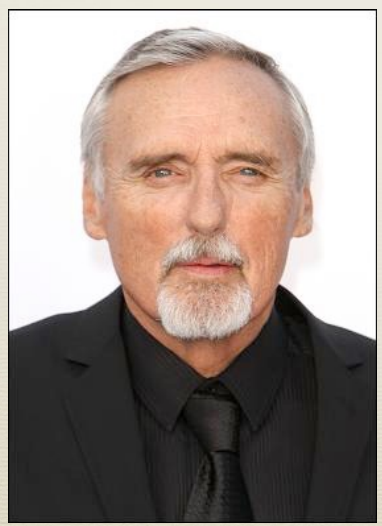
Samuel Jackson



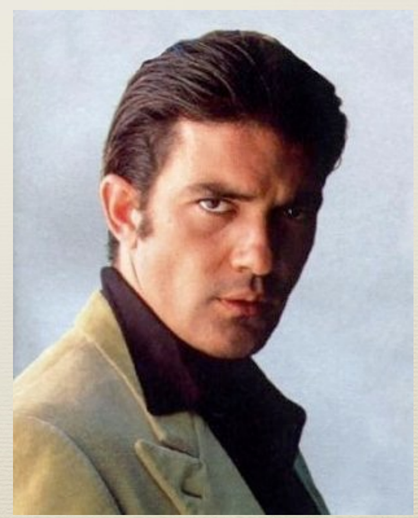
Sharon Stone

Tom Hanks

Martin Sheen



Dennis Hopper



Antonio Banderas

Madonna

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$$\sum_{t>0} \frac{1}{t} \left| \{ y \mid d(y,x) = t \} \right|$$

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- Sampling: a fraction of breadth-first visits, very unreliable results on graphs that are not strongly connected, needs direct access
- Edith Cohen's [JCSS 1997] size estimation framework: very powerful but does not scale or parallelize really well, needs direct

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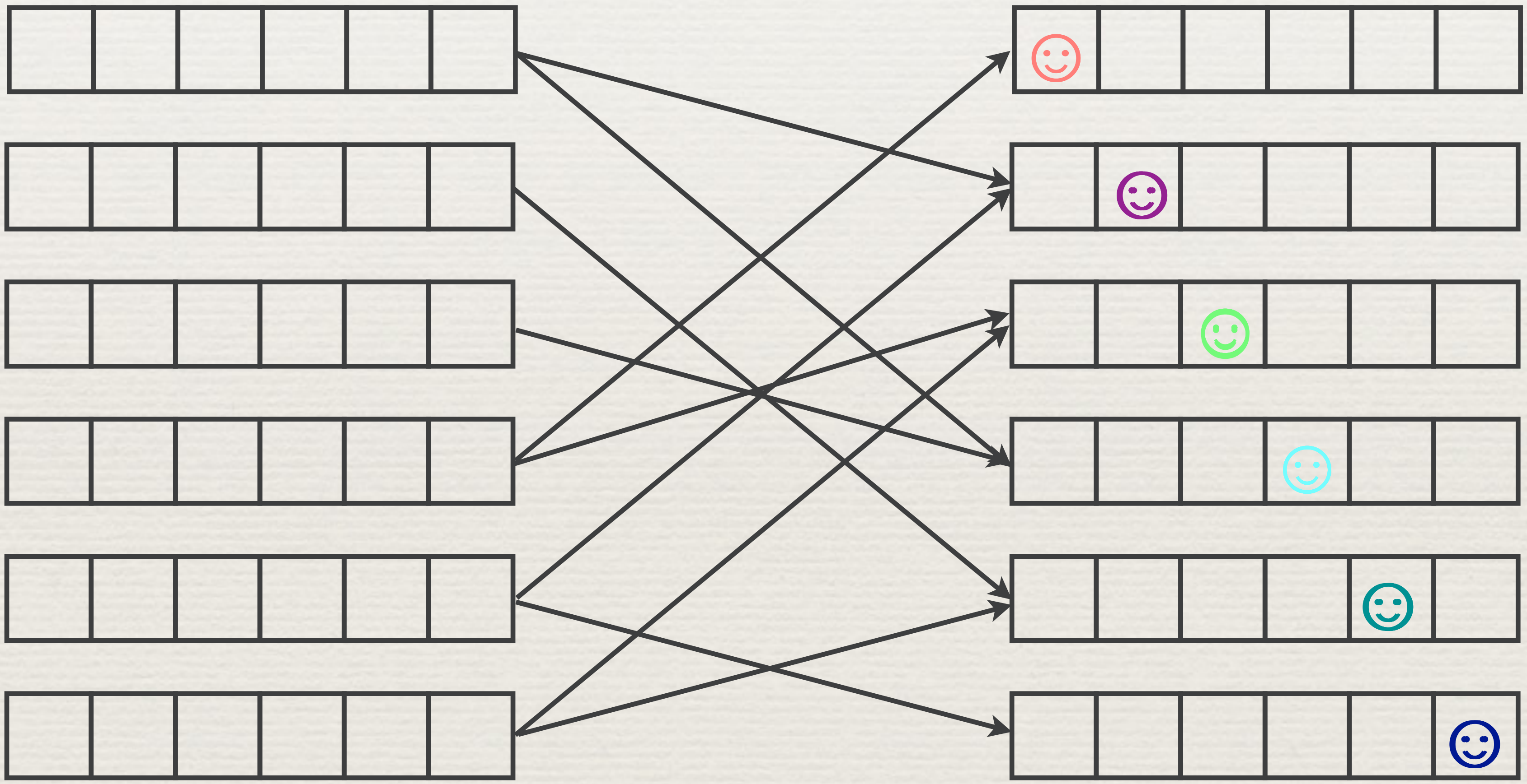
- But also $B_{t+1}(x)=\bigcup_{x\to y}B_t(y)\cup\{x\}$
- So we can compute balls by enumerating the arcs $x\to y$ and performing set unions

A round of updates



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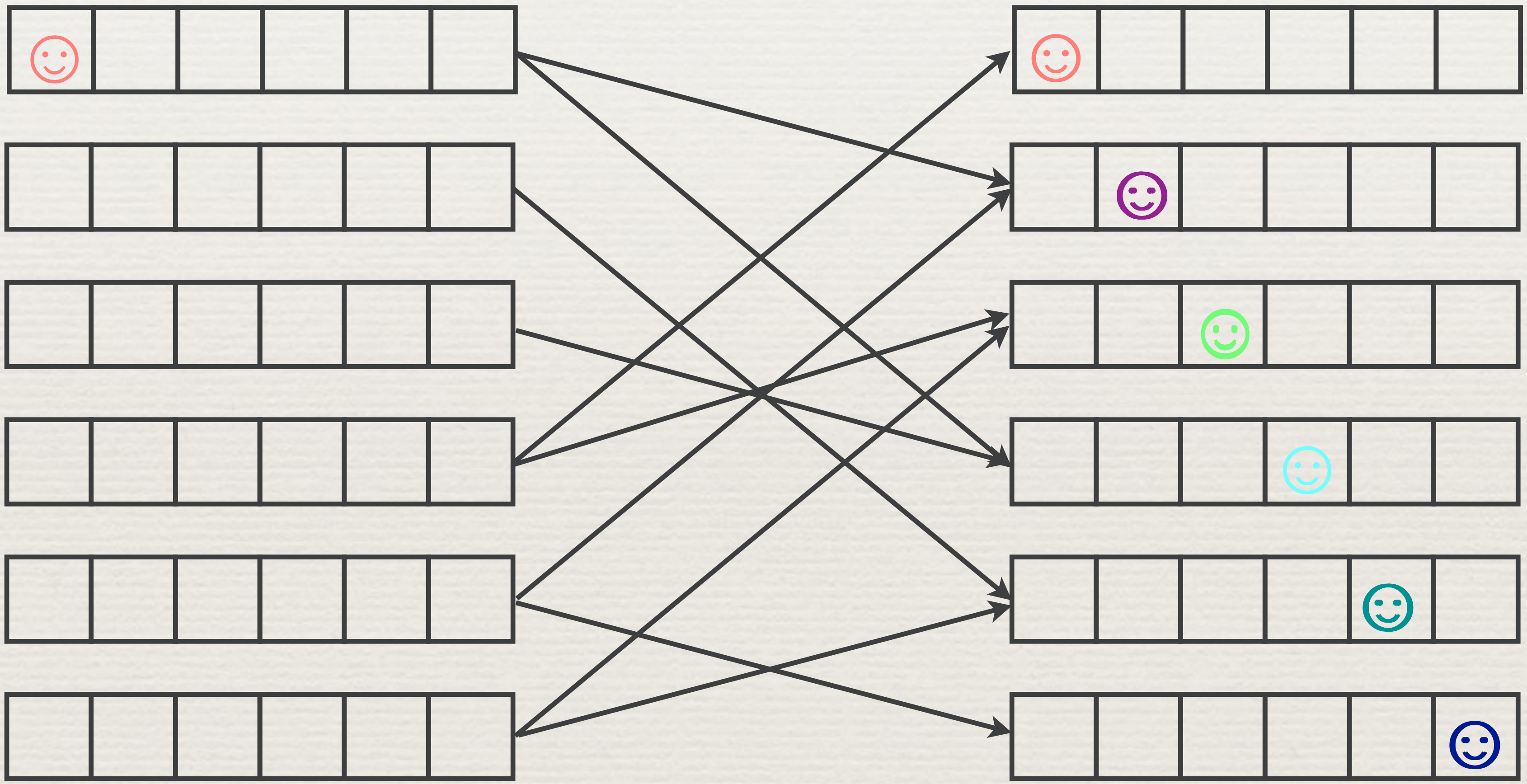

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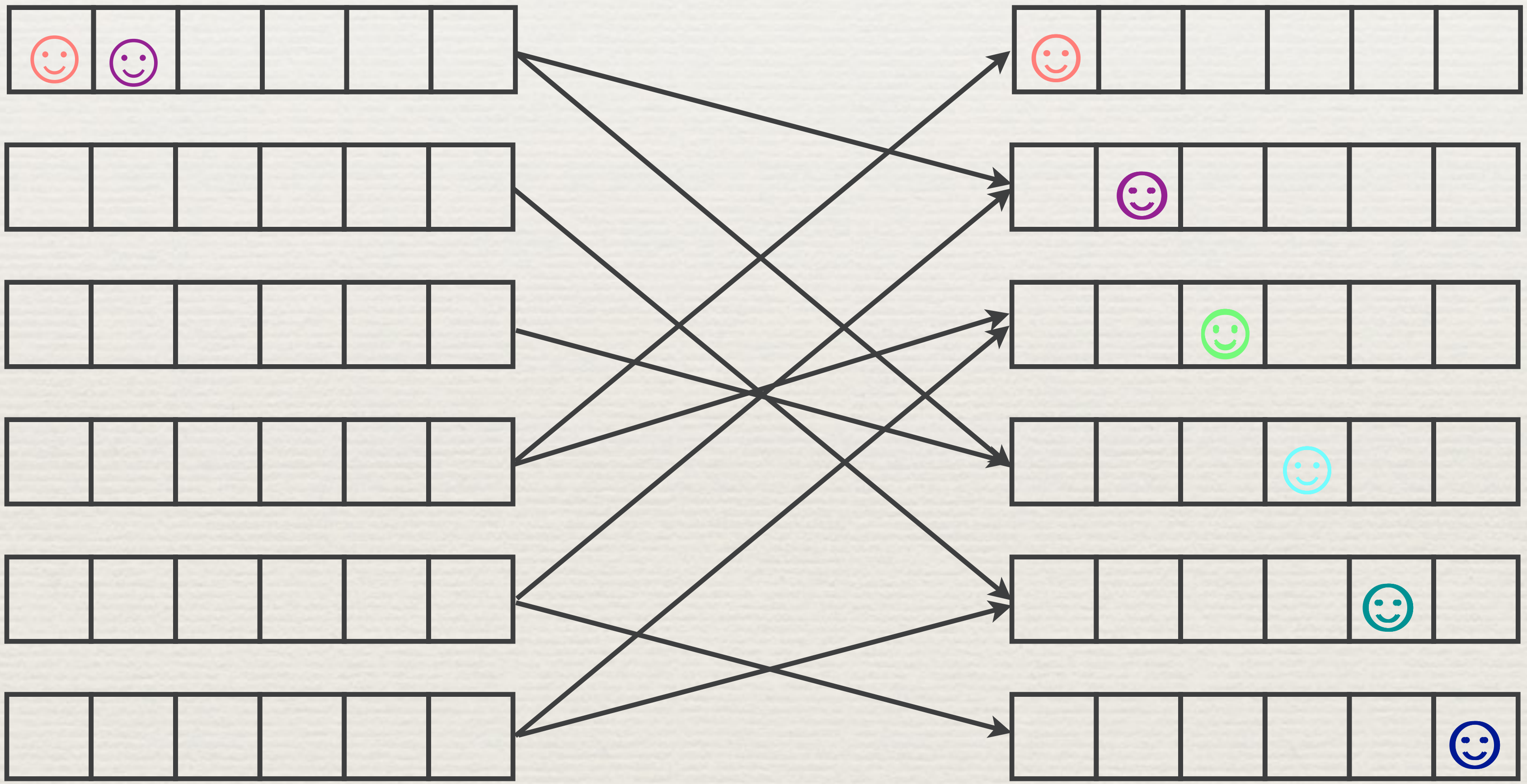

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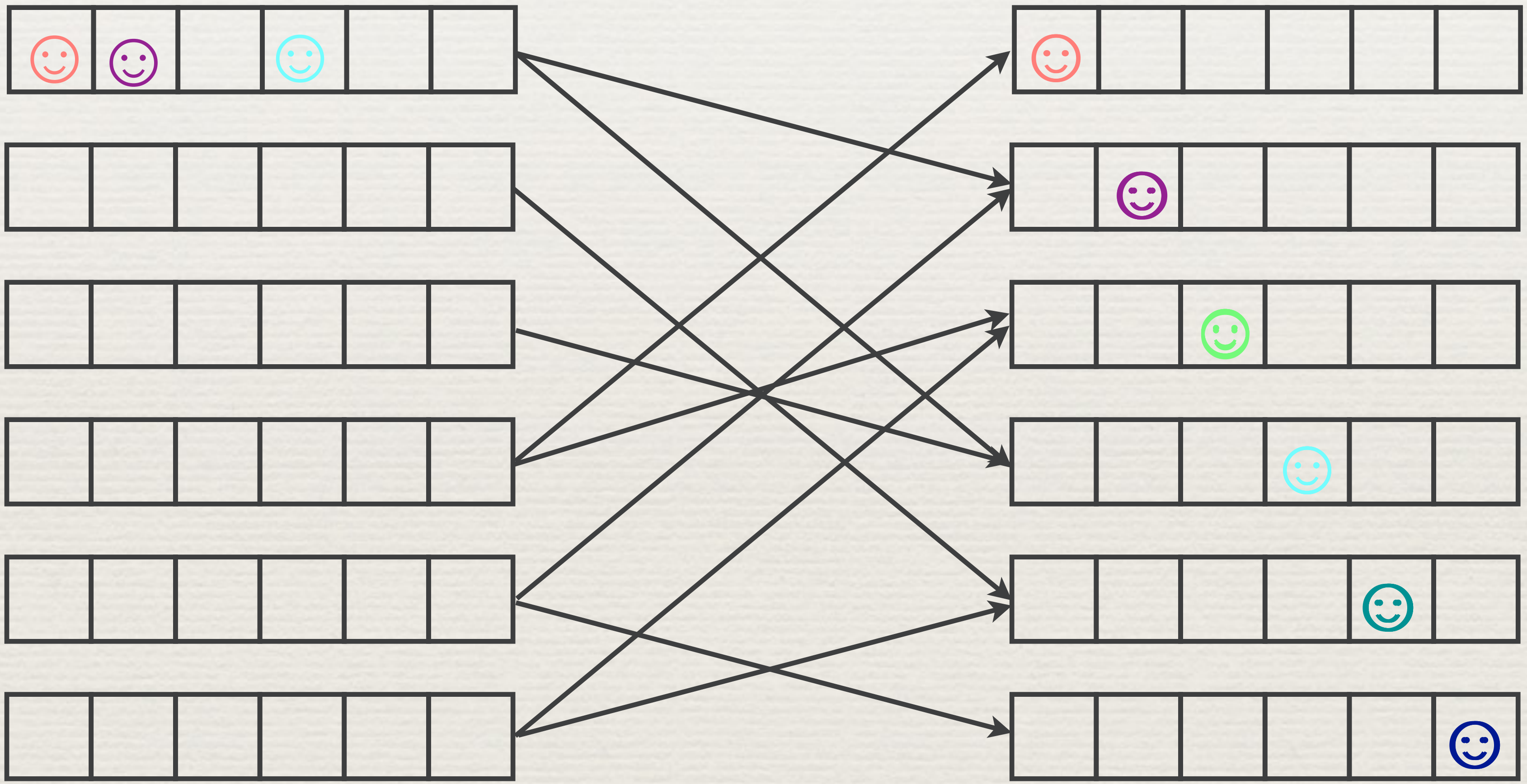

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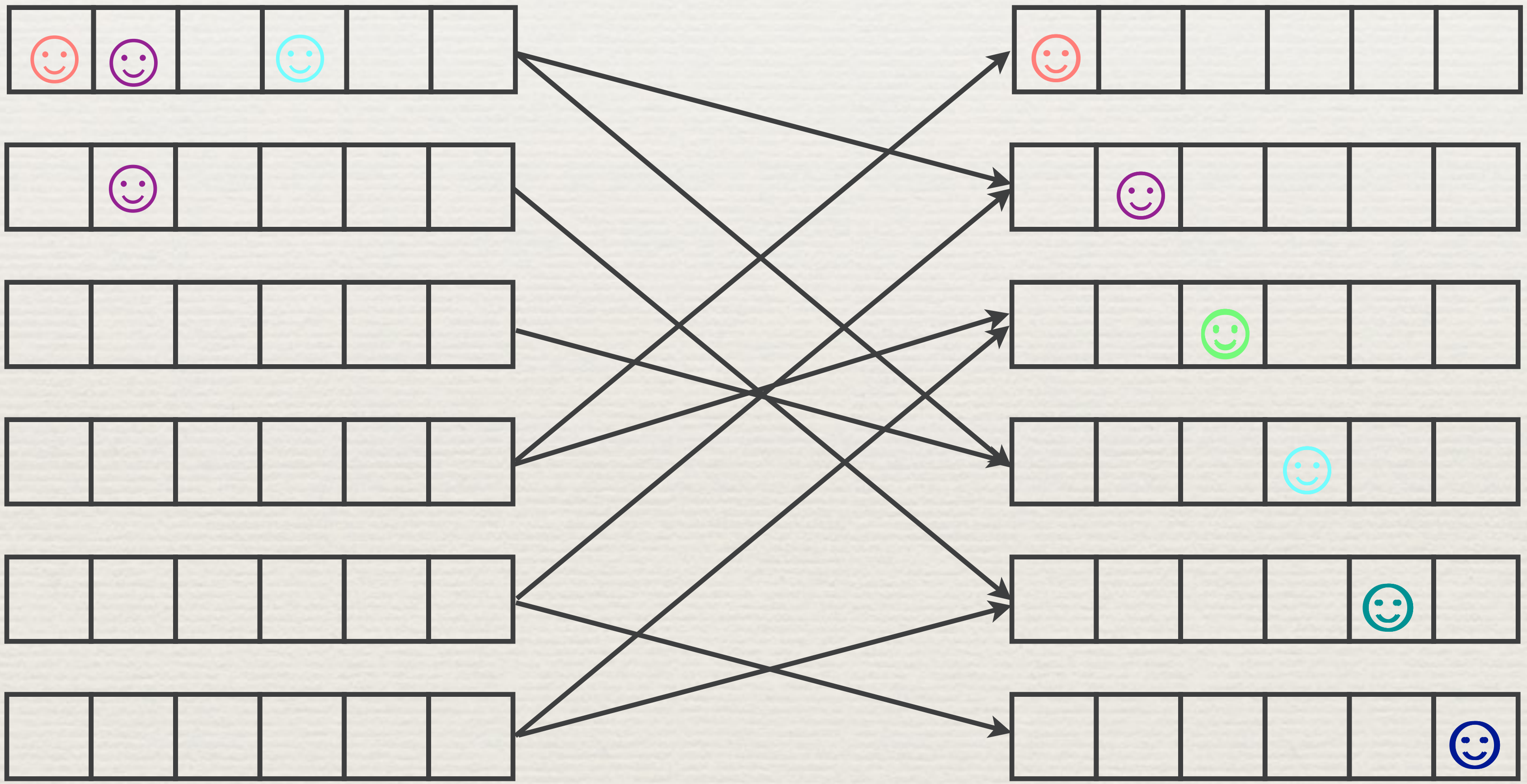

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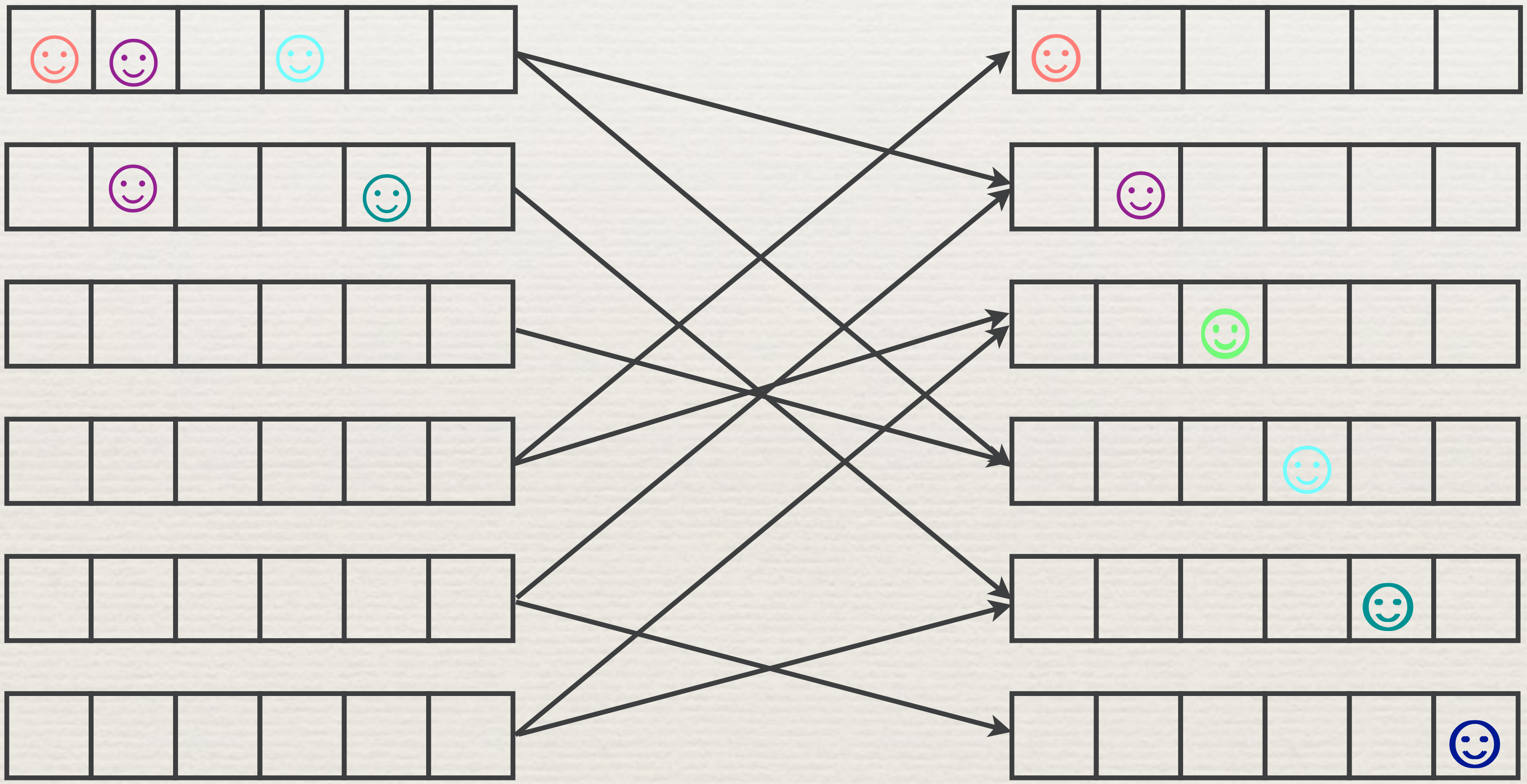

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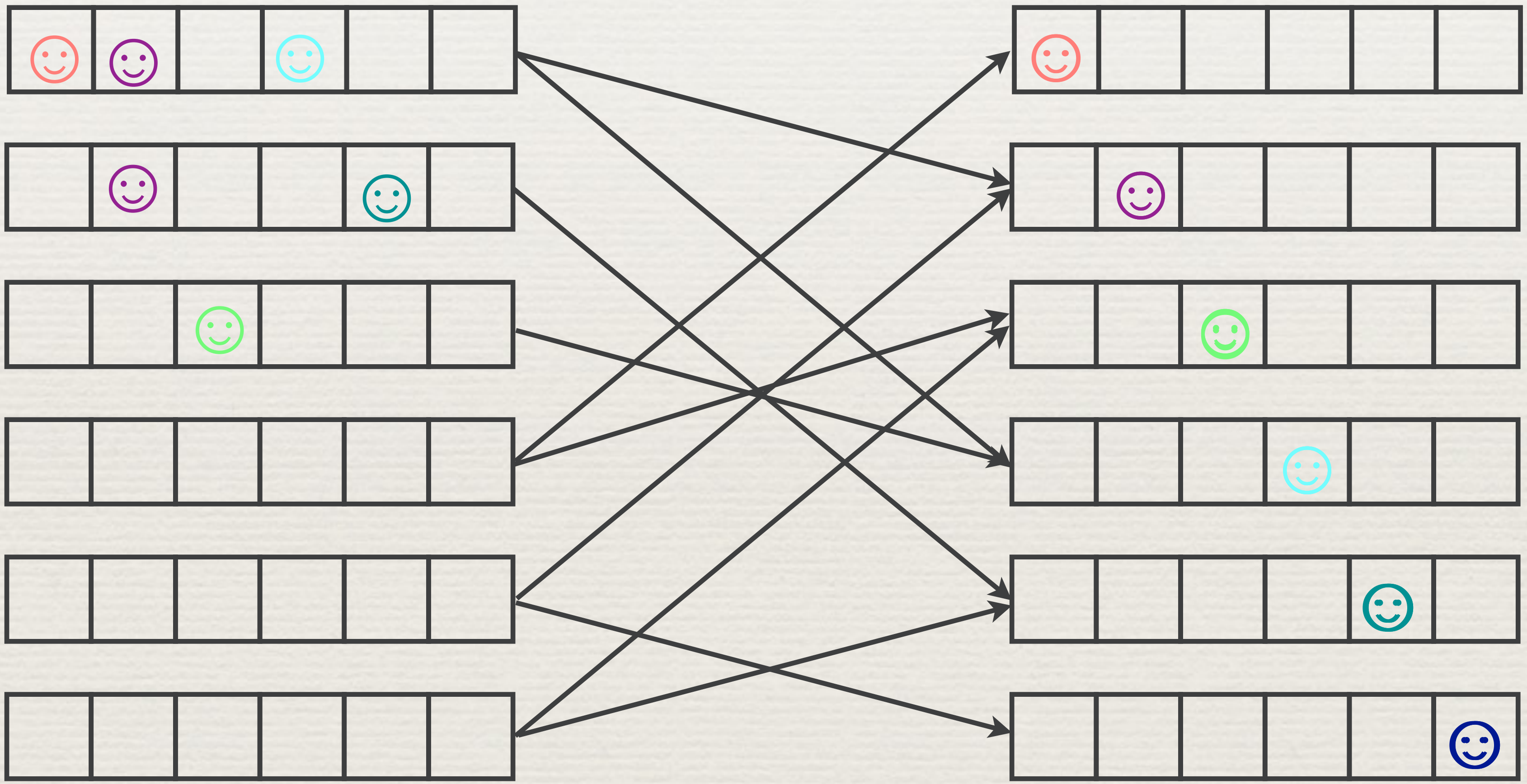

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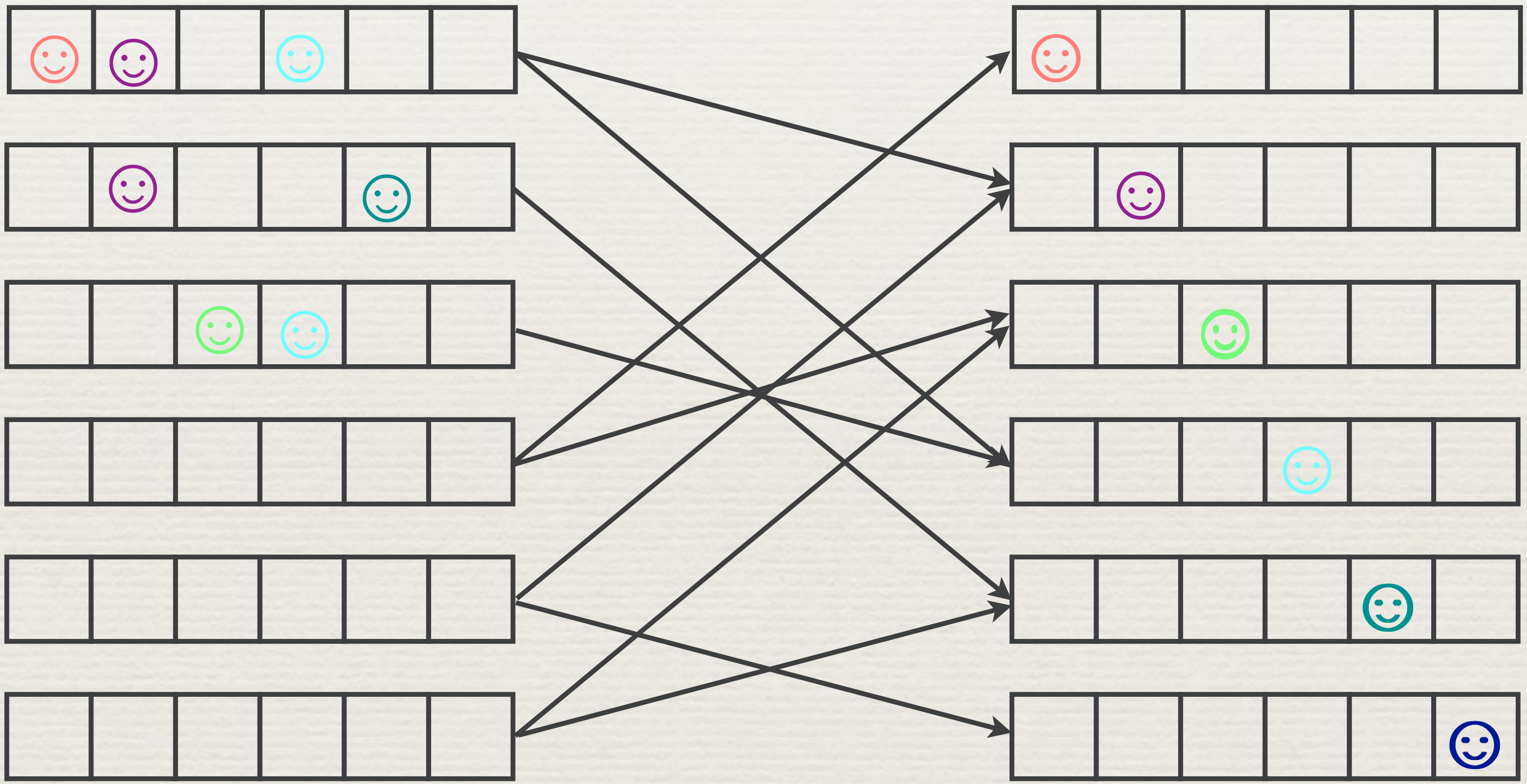

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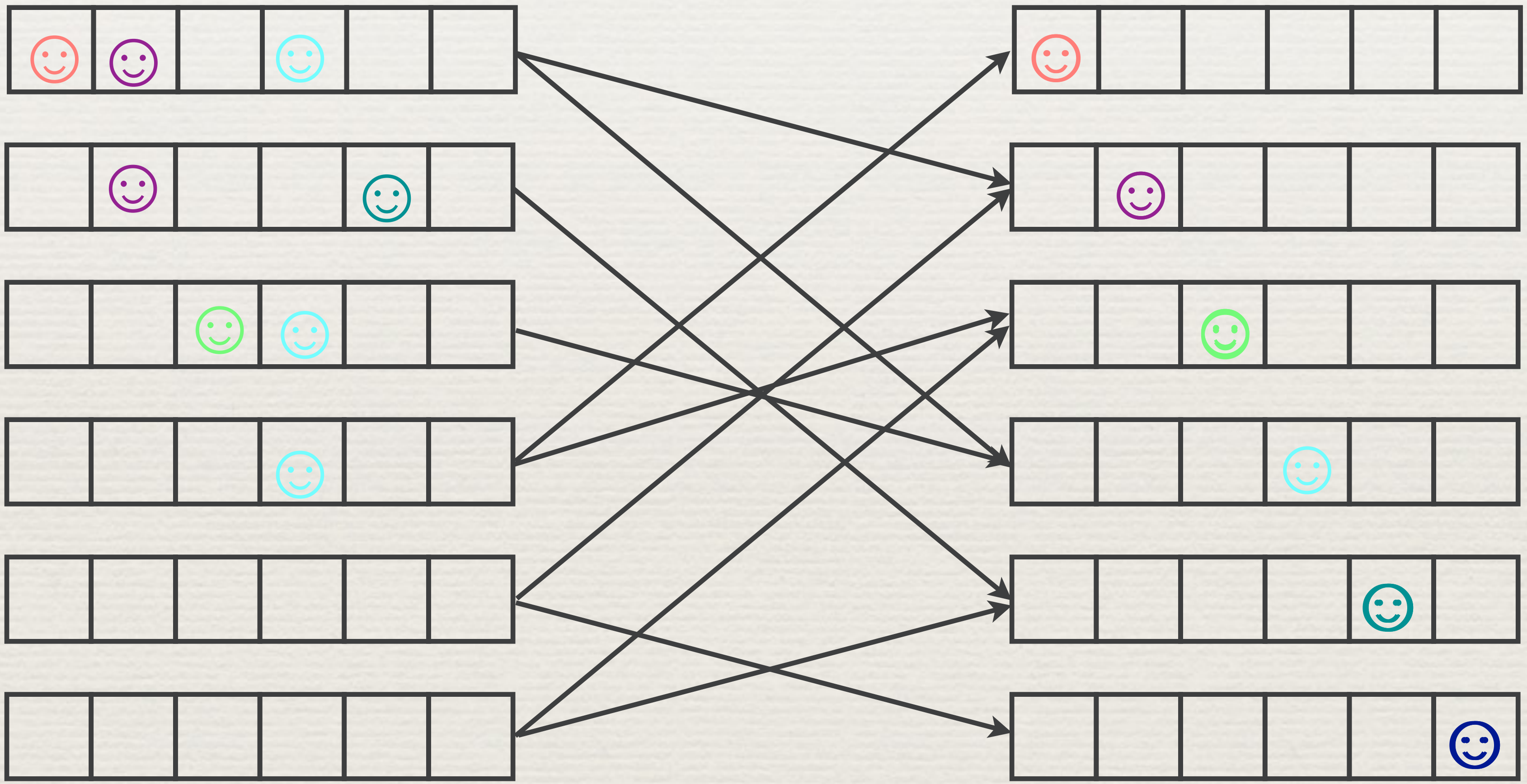

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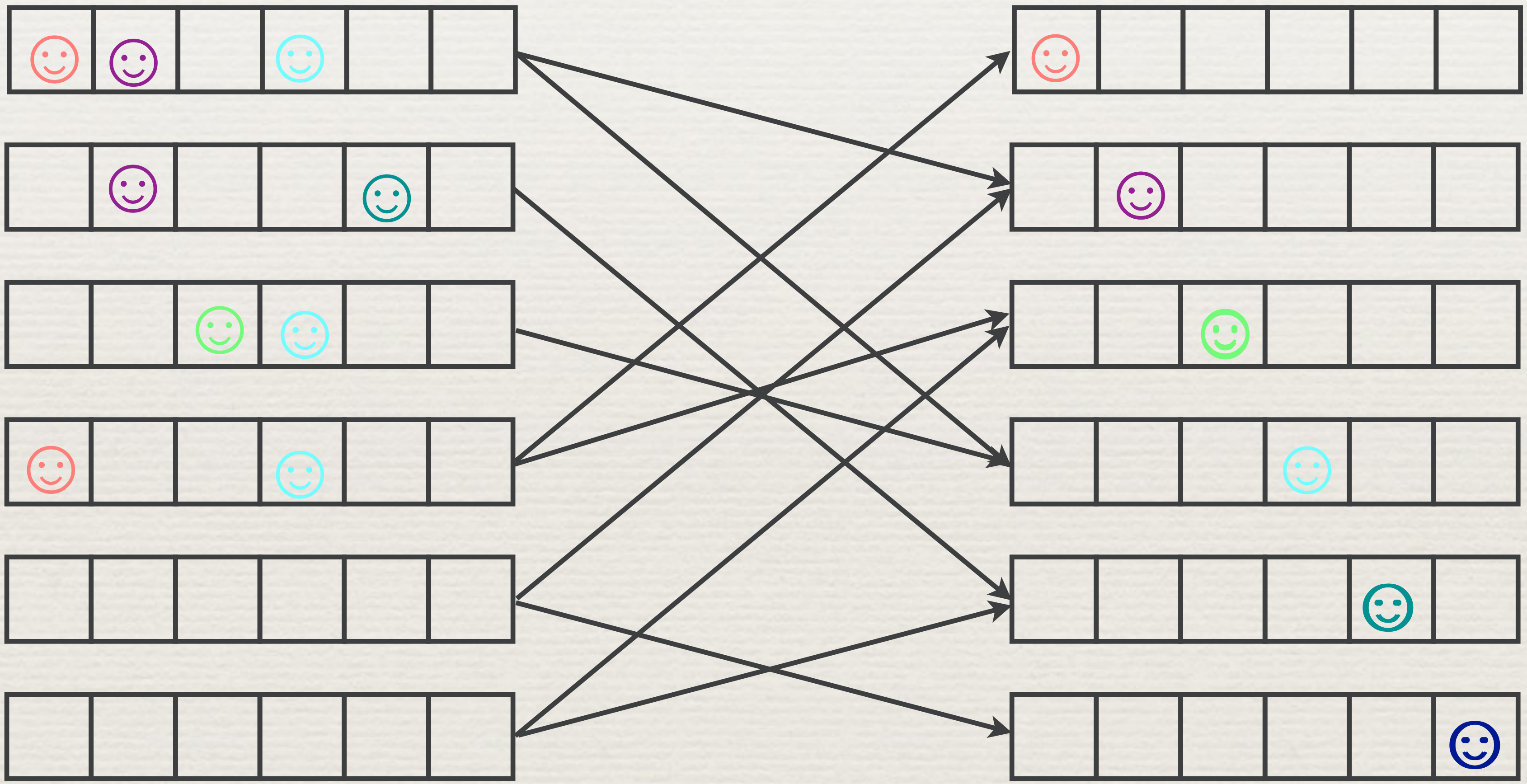

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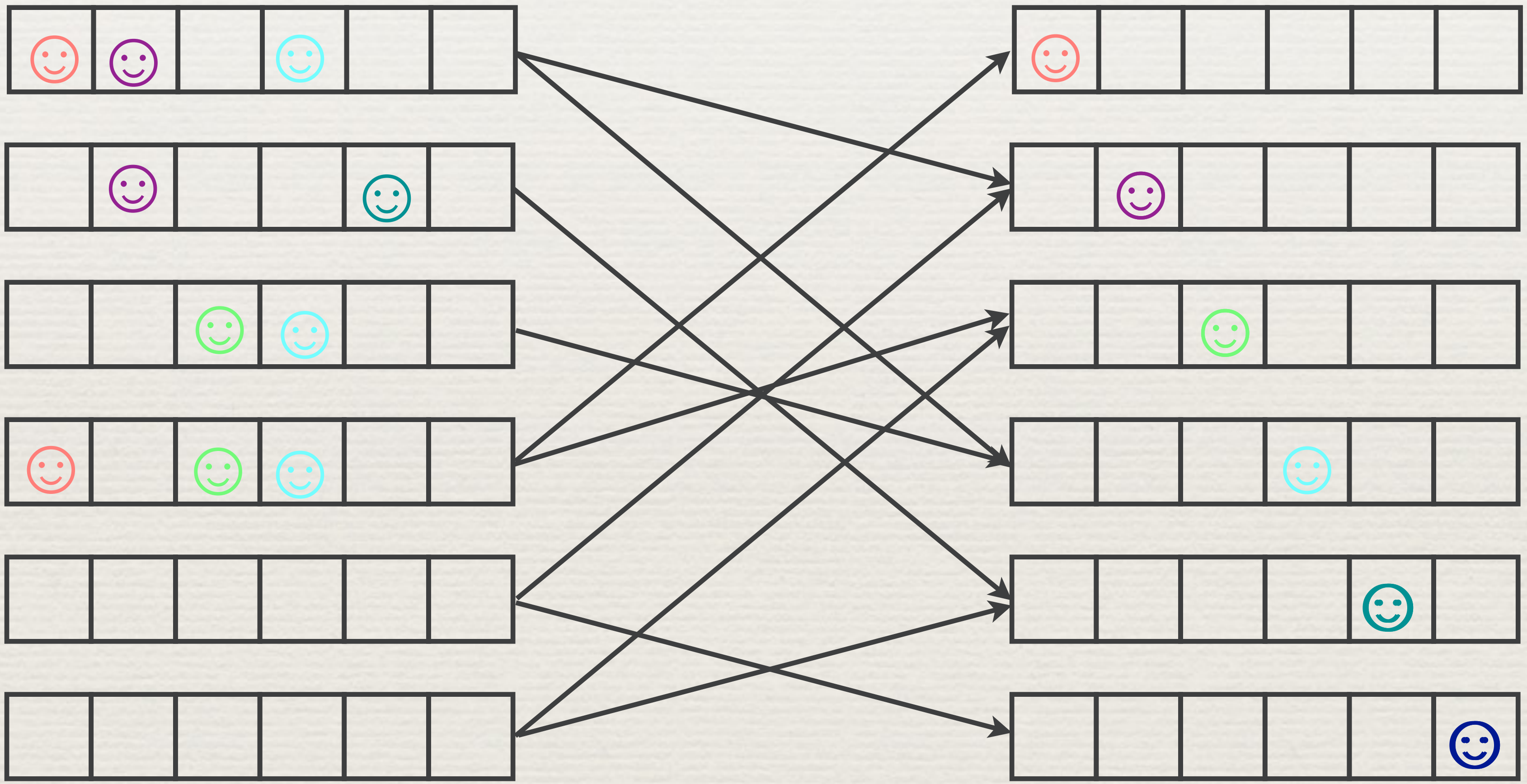

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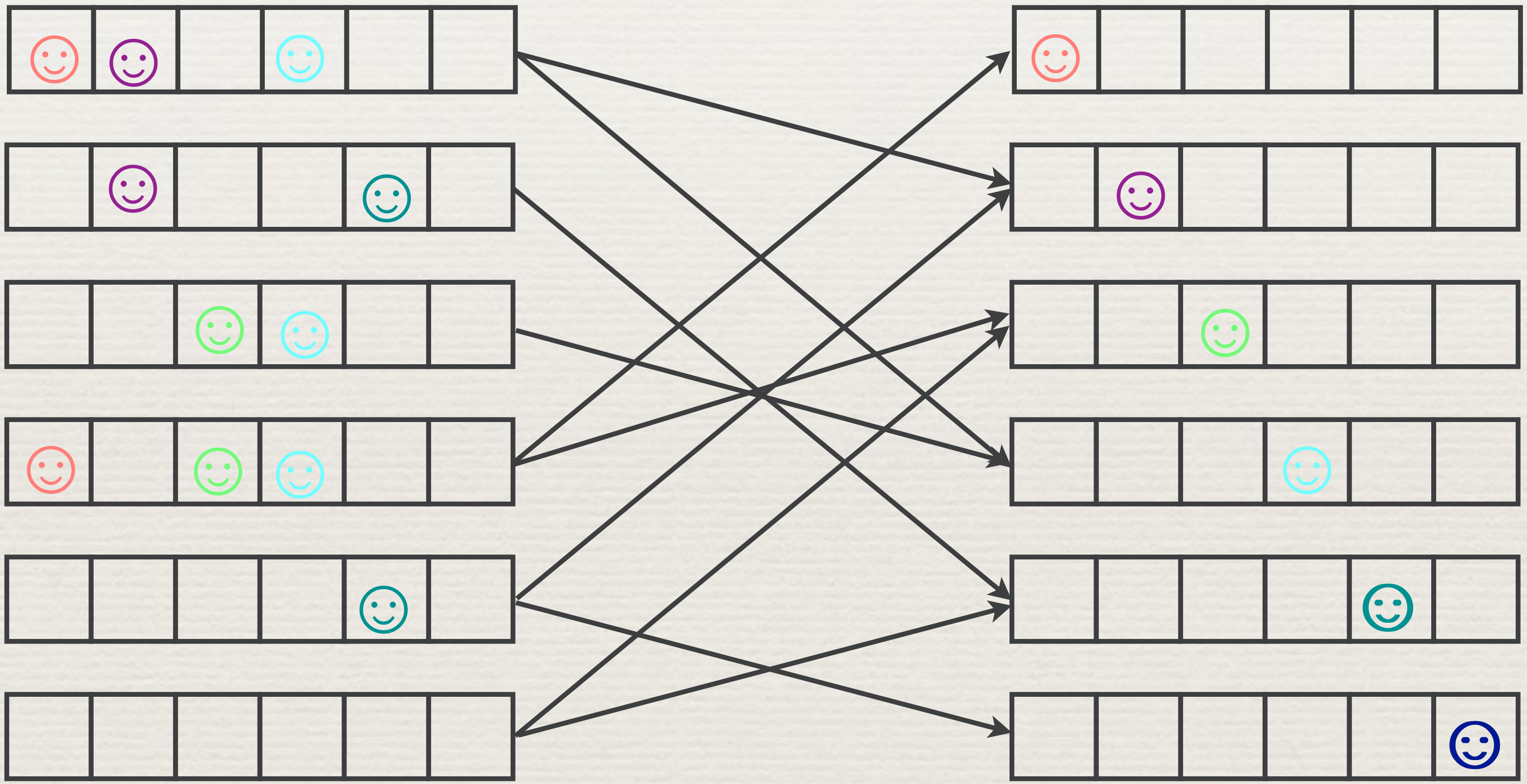

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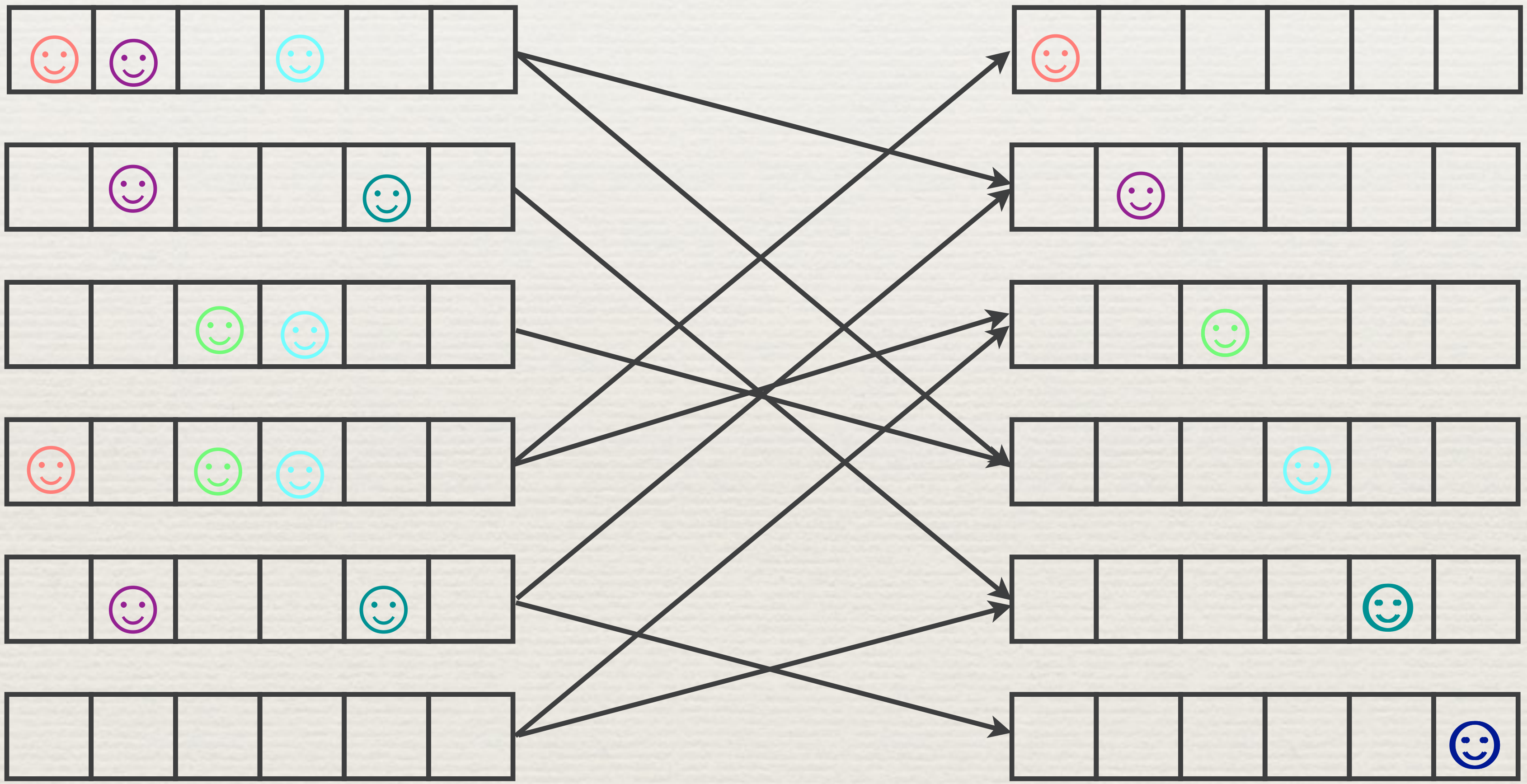

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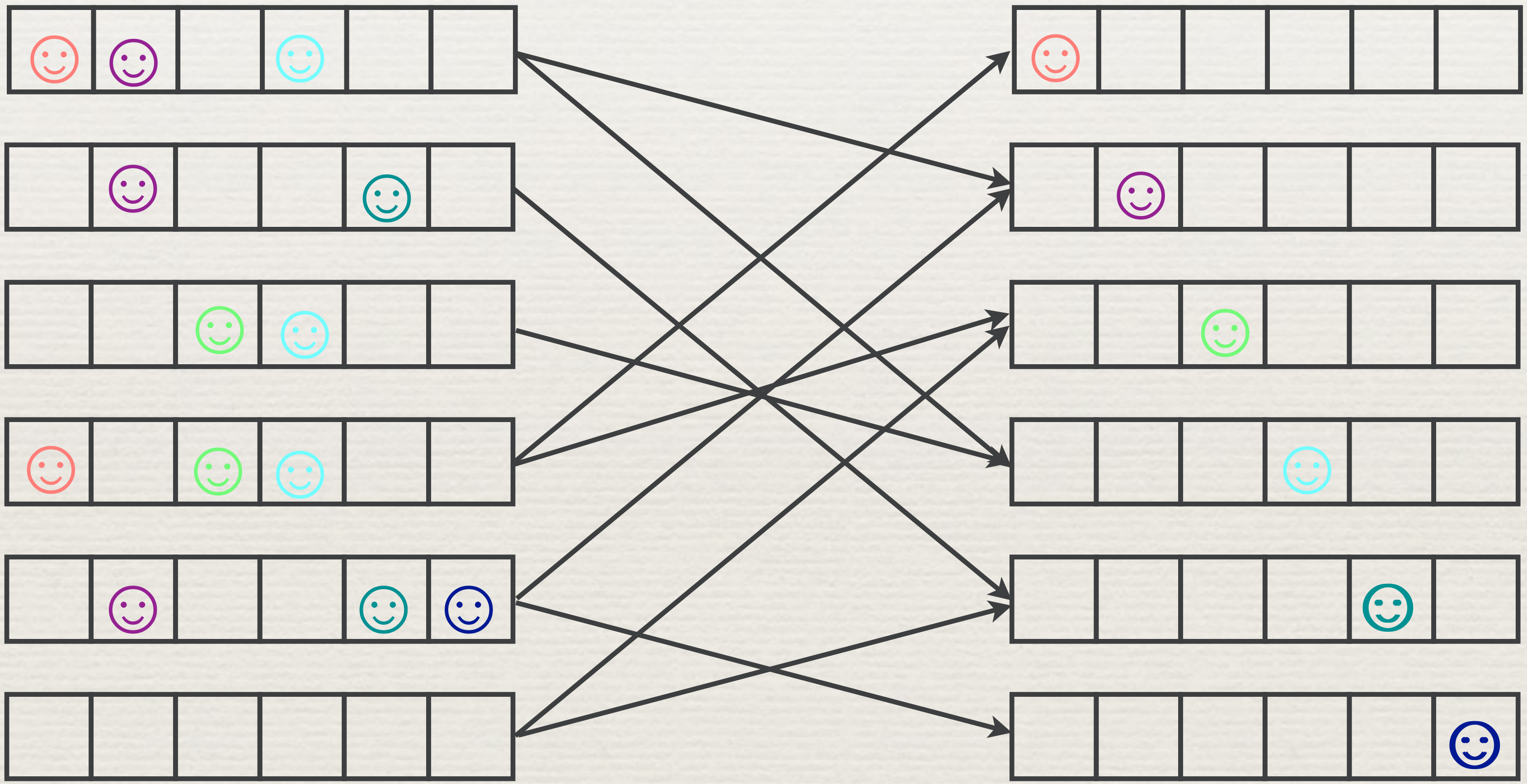

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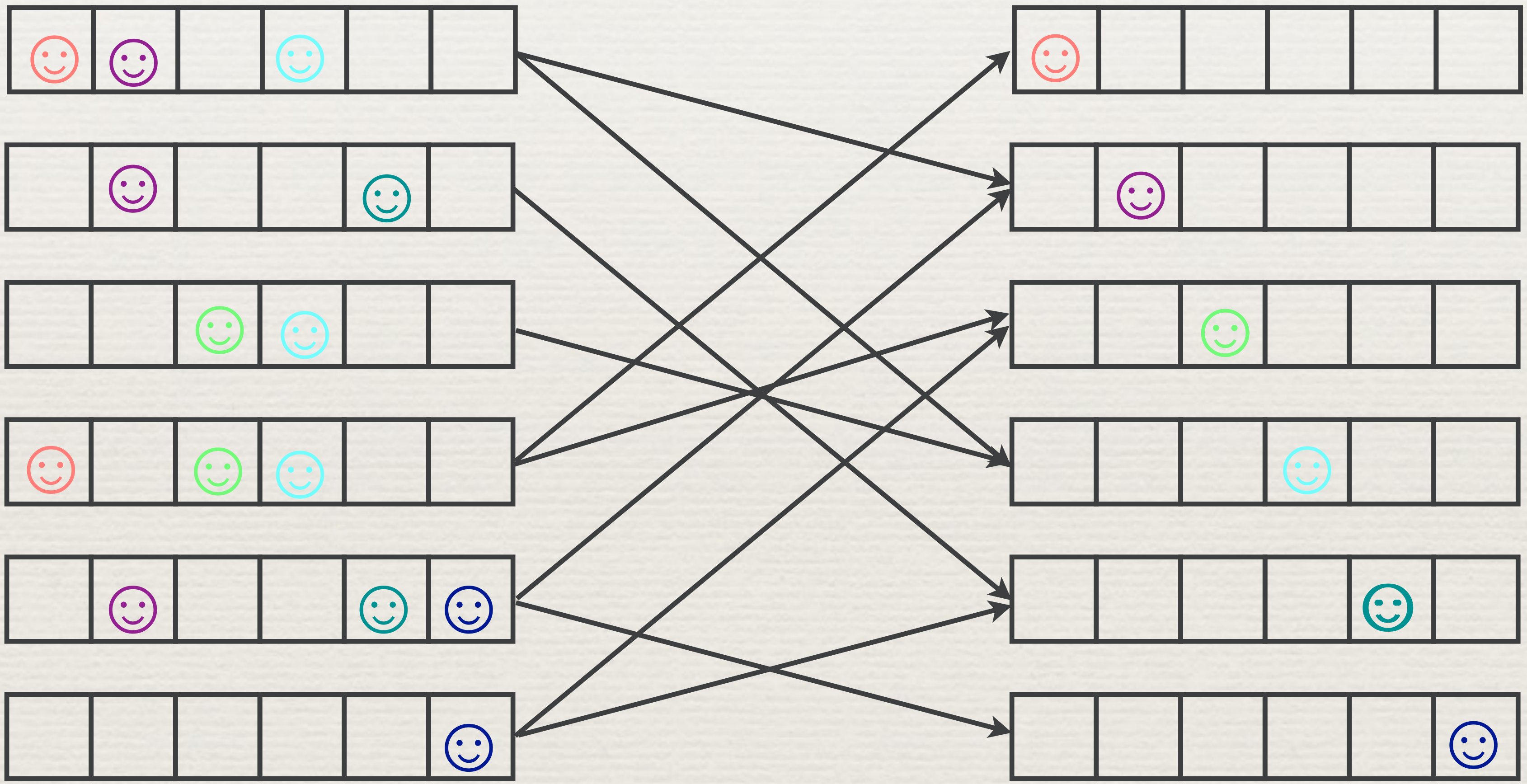

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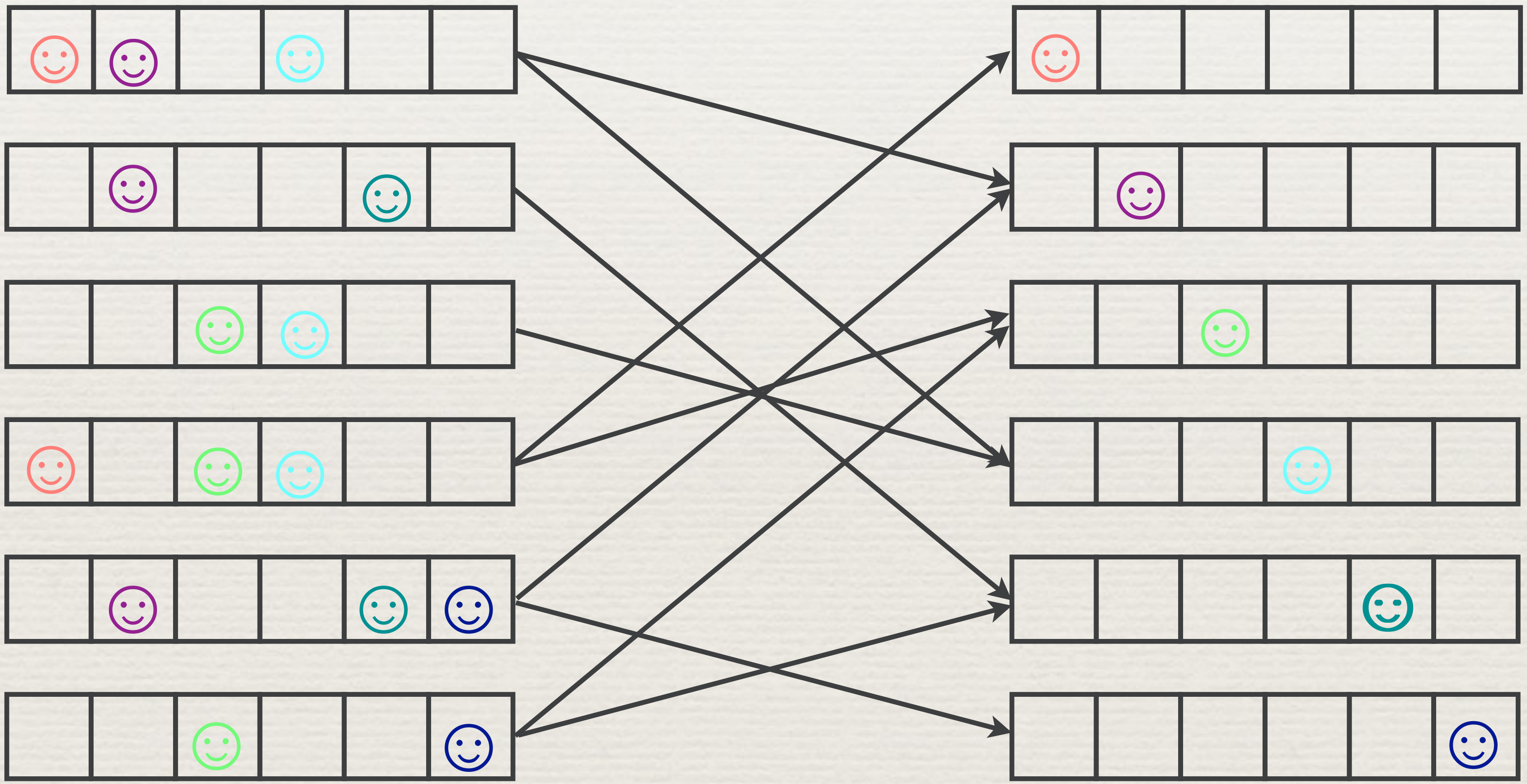

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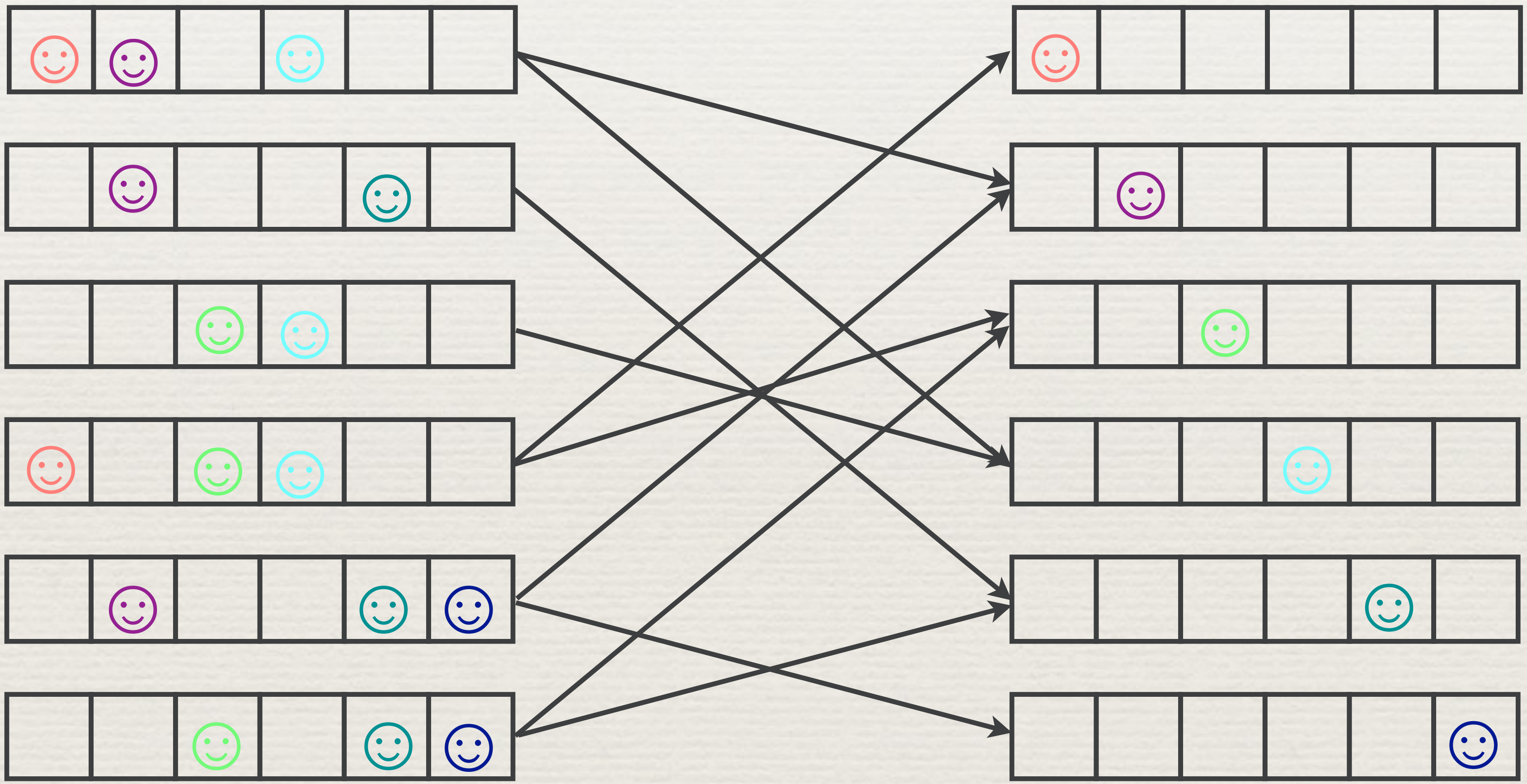

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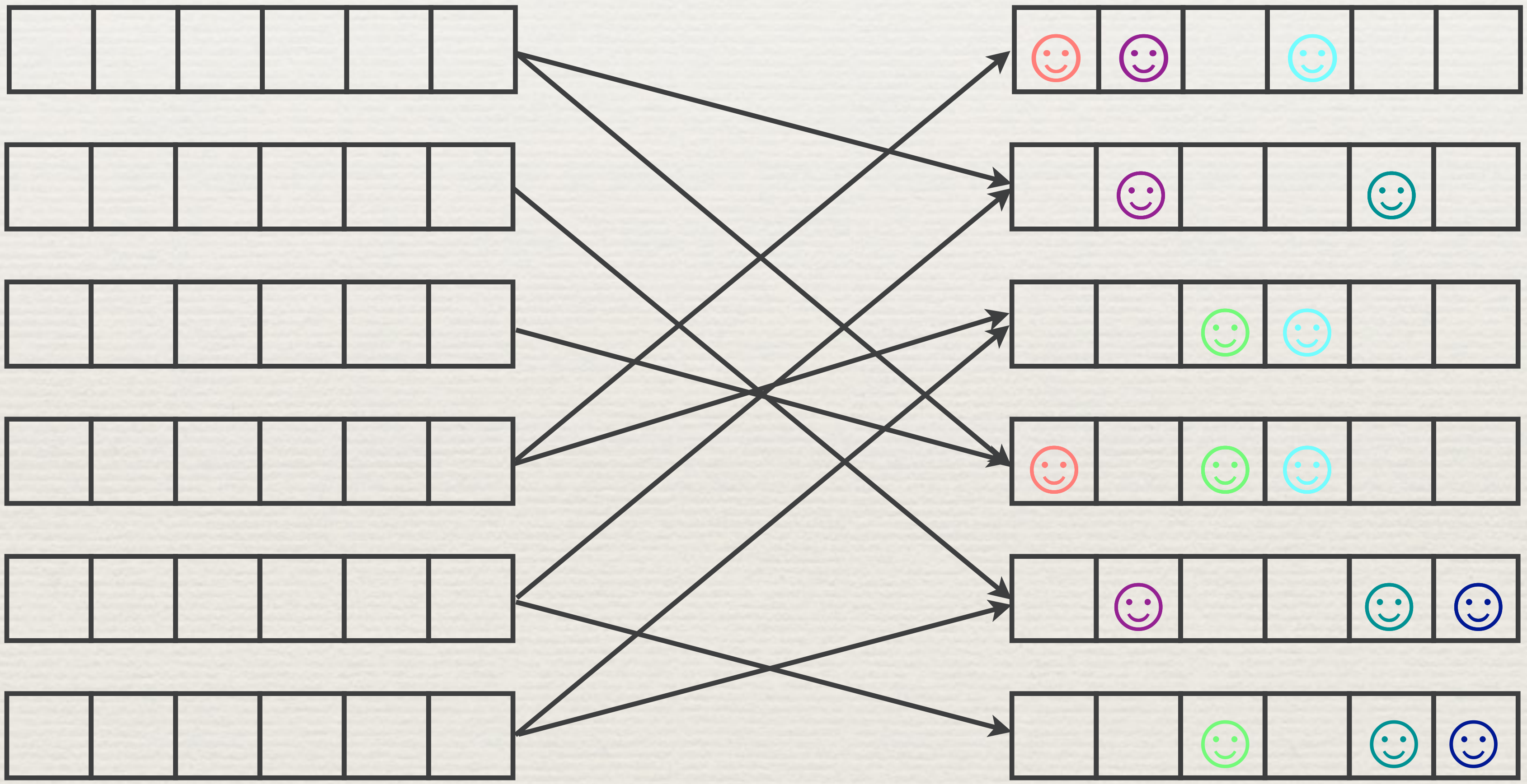

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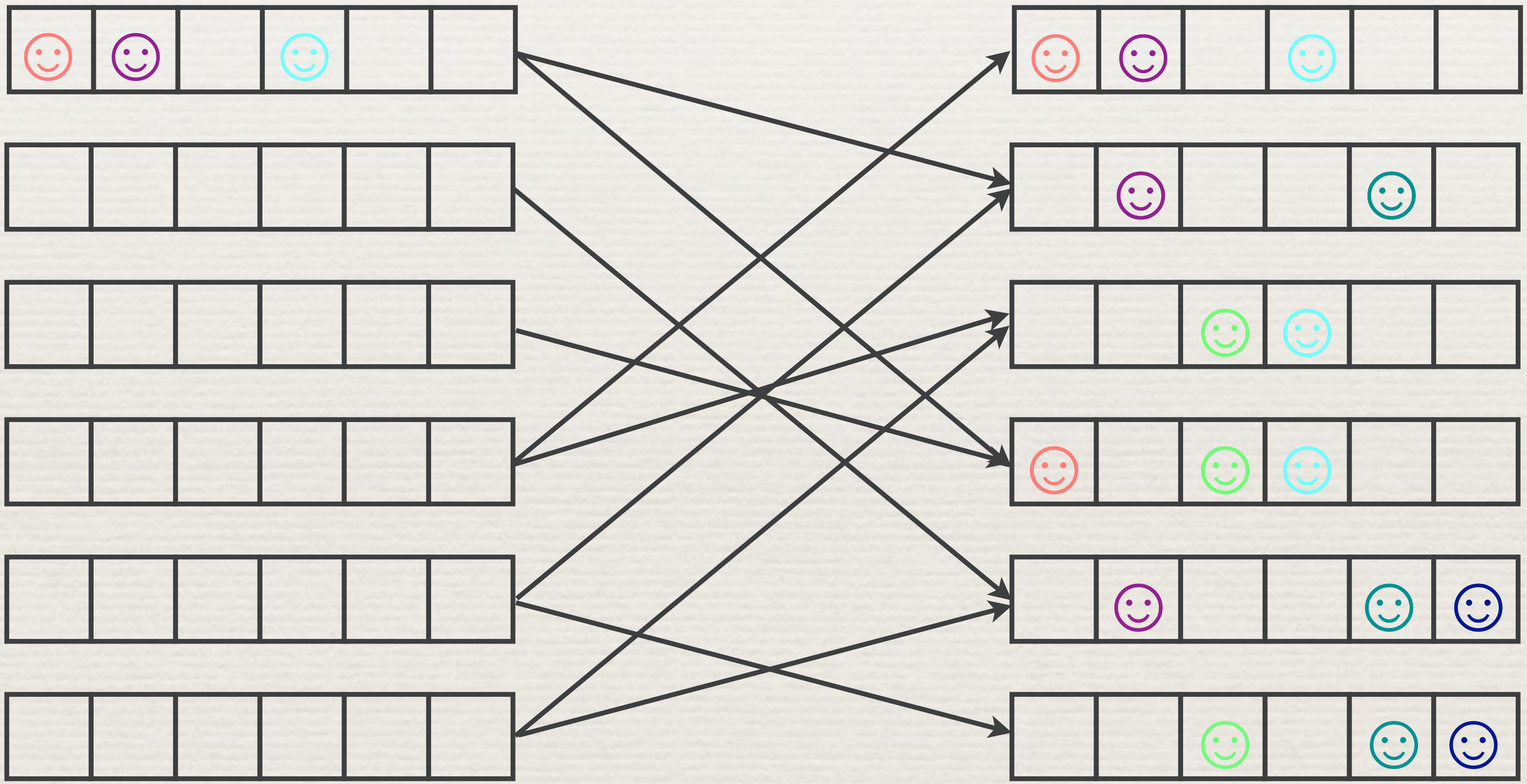

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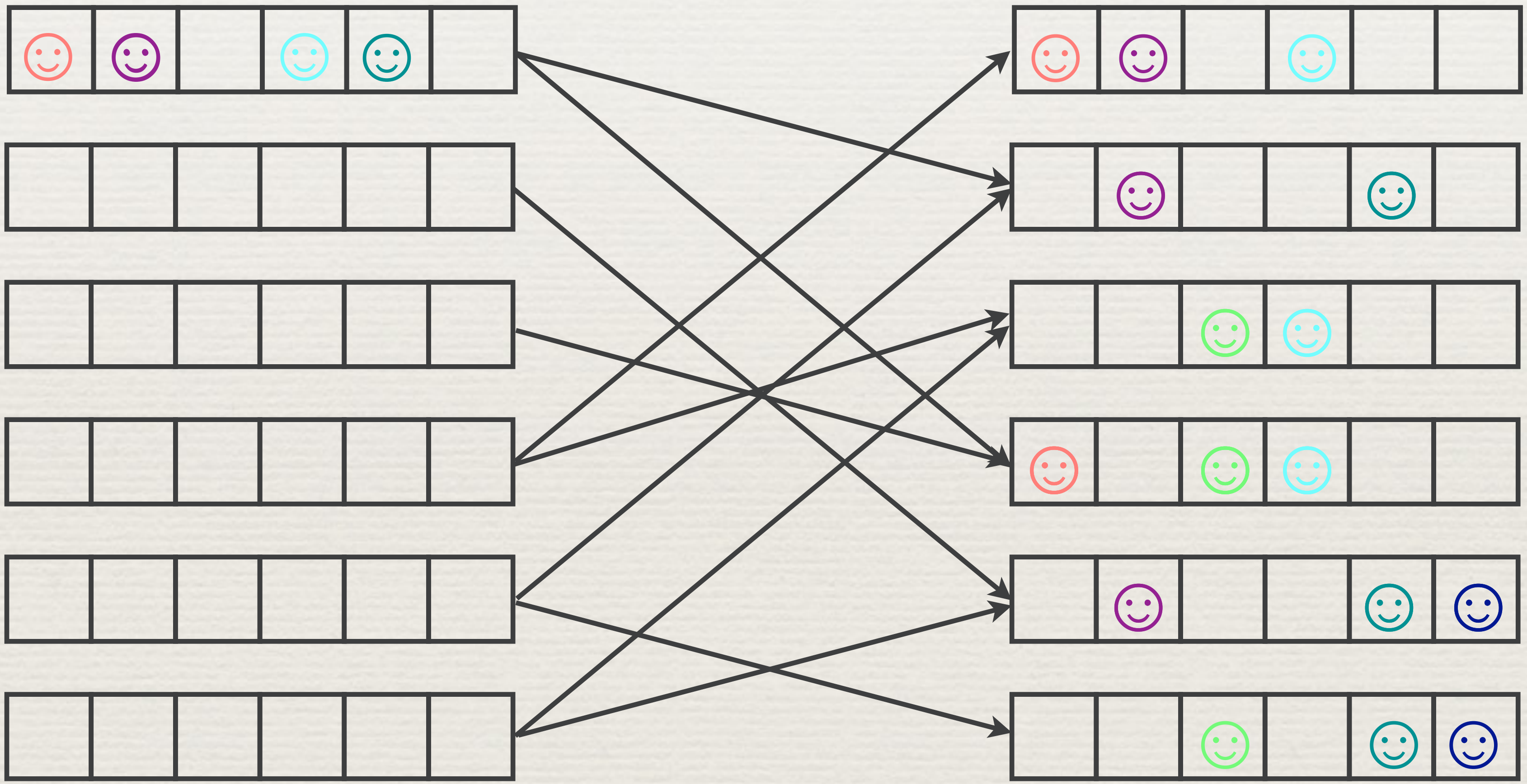

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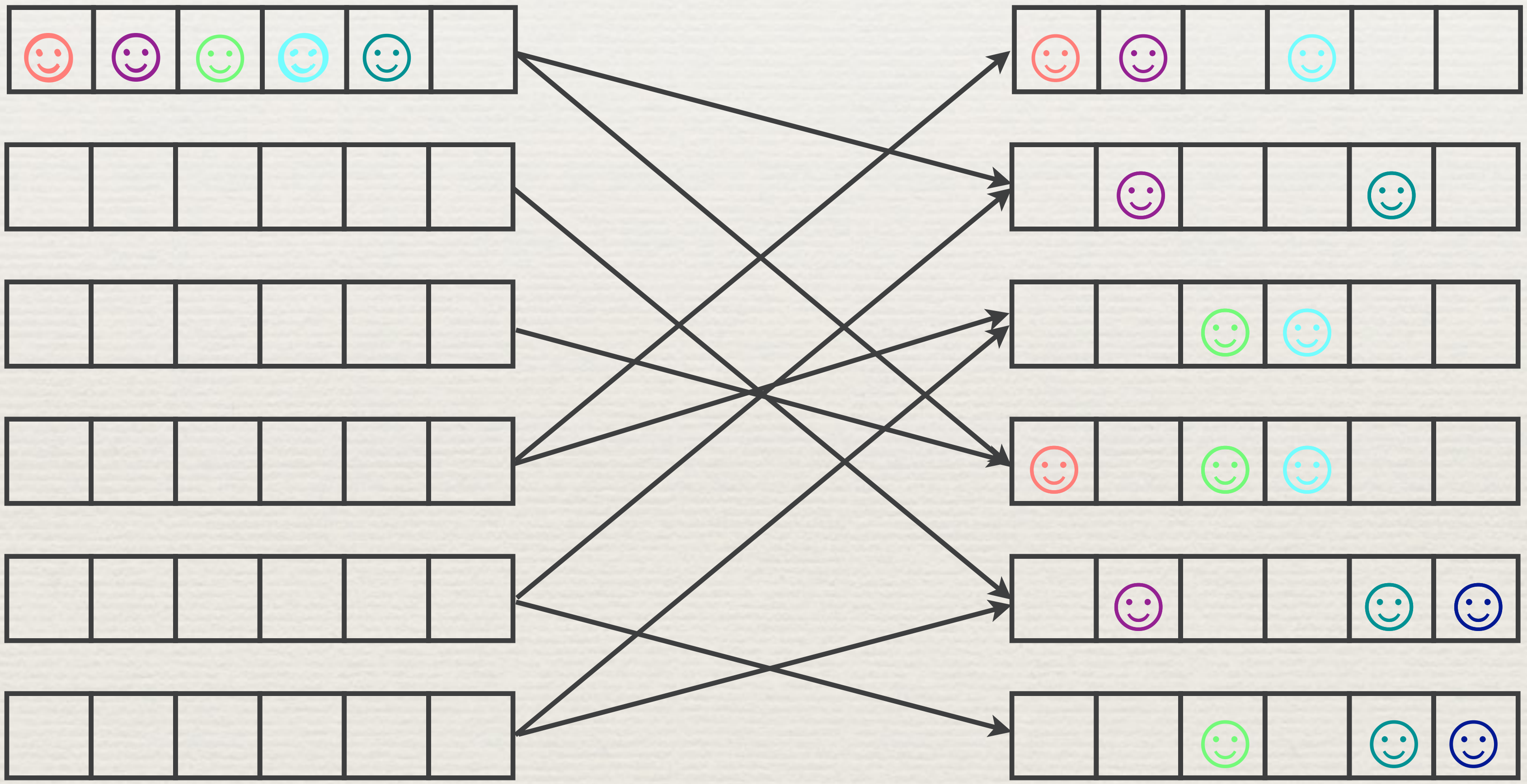

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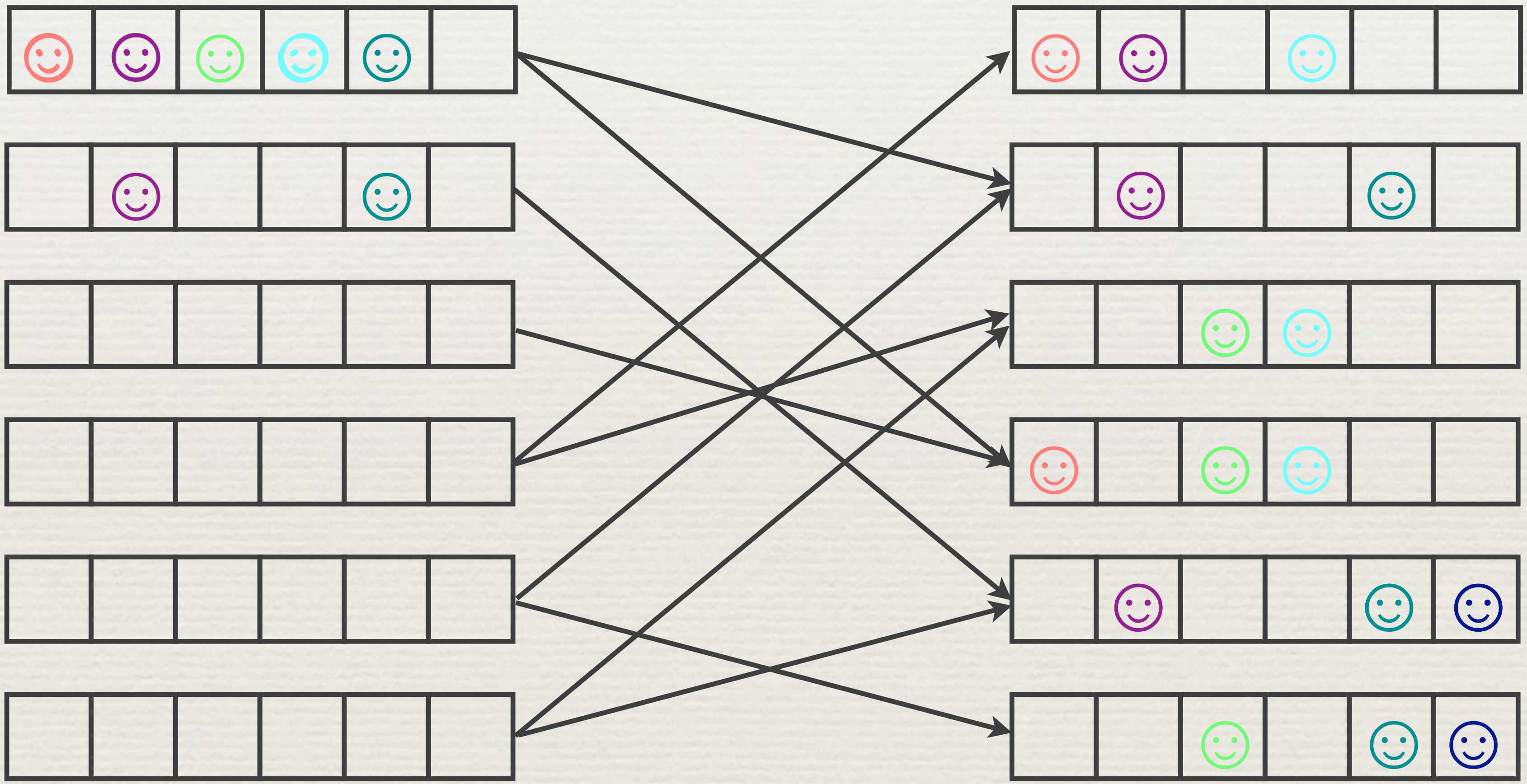

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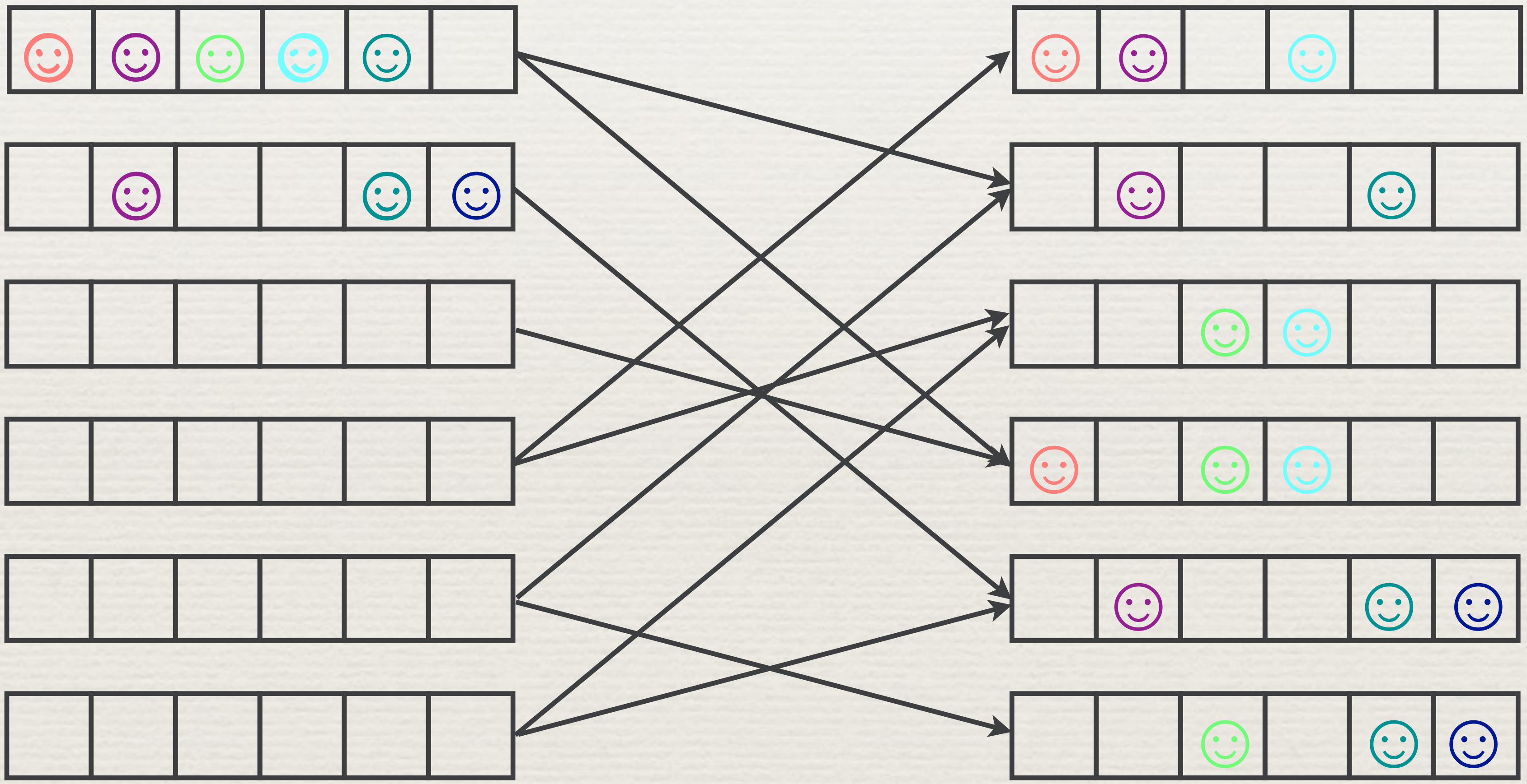

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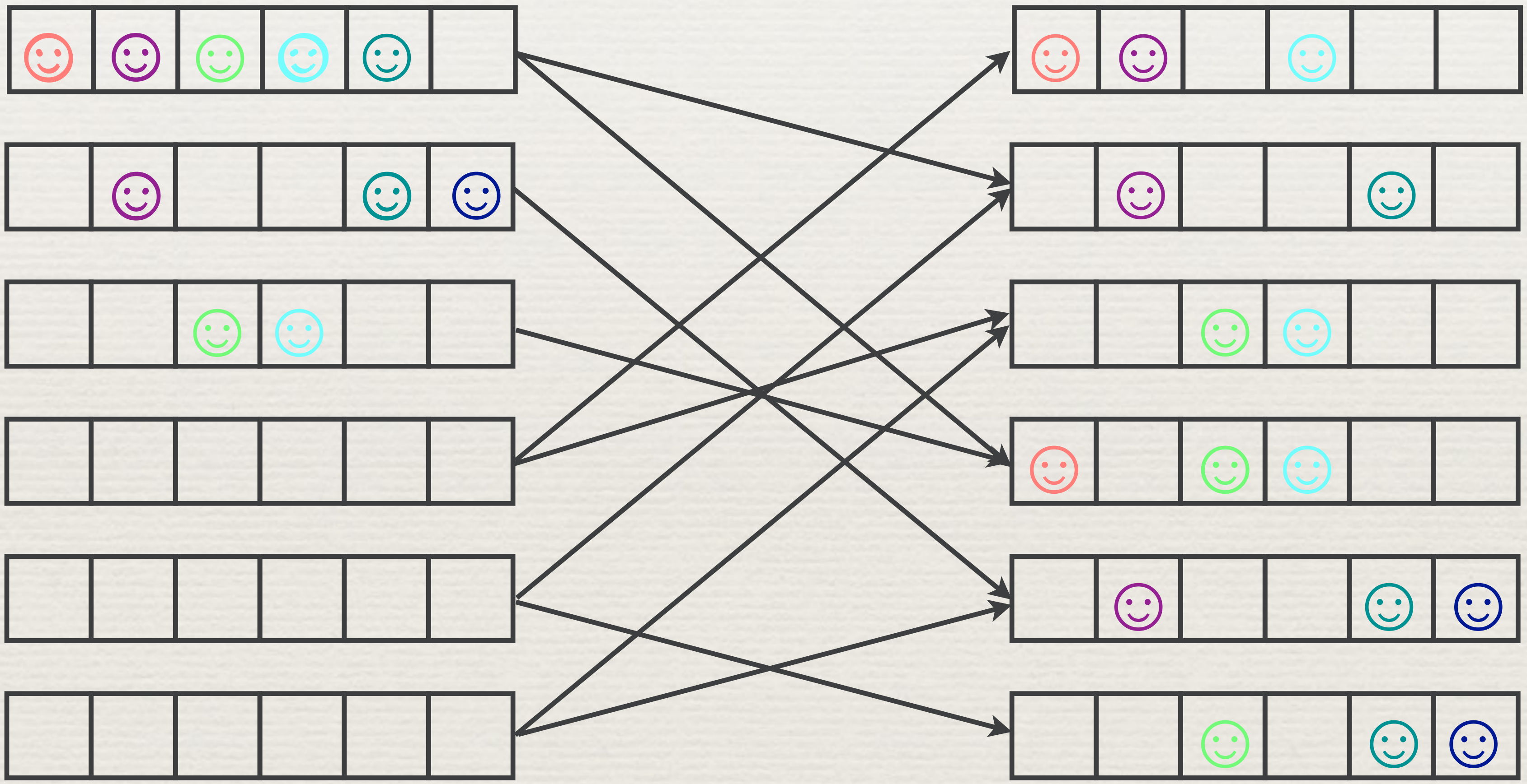

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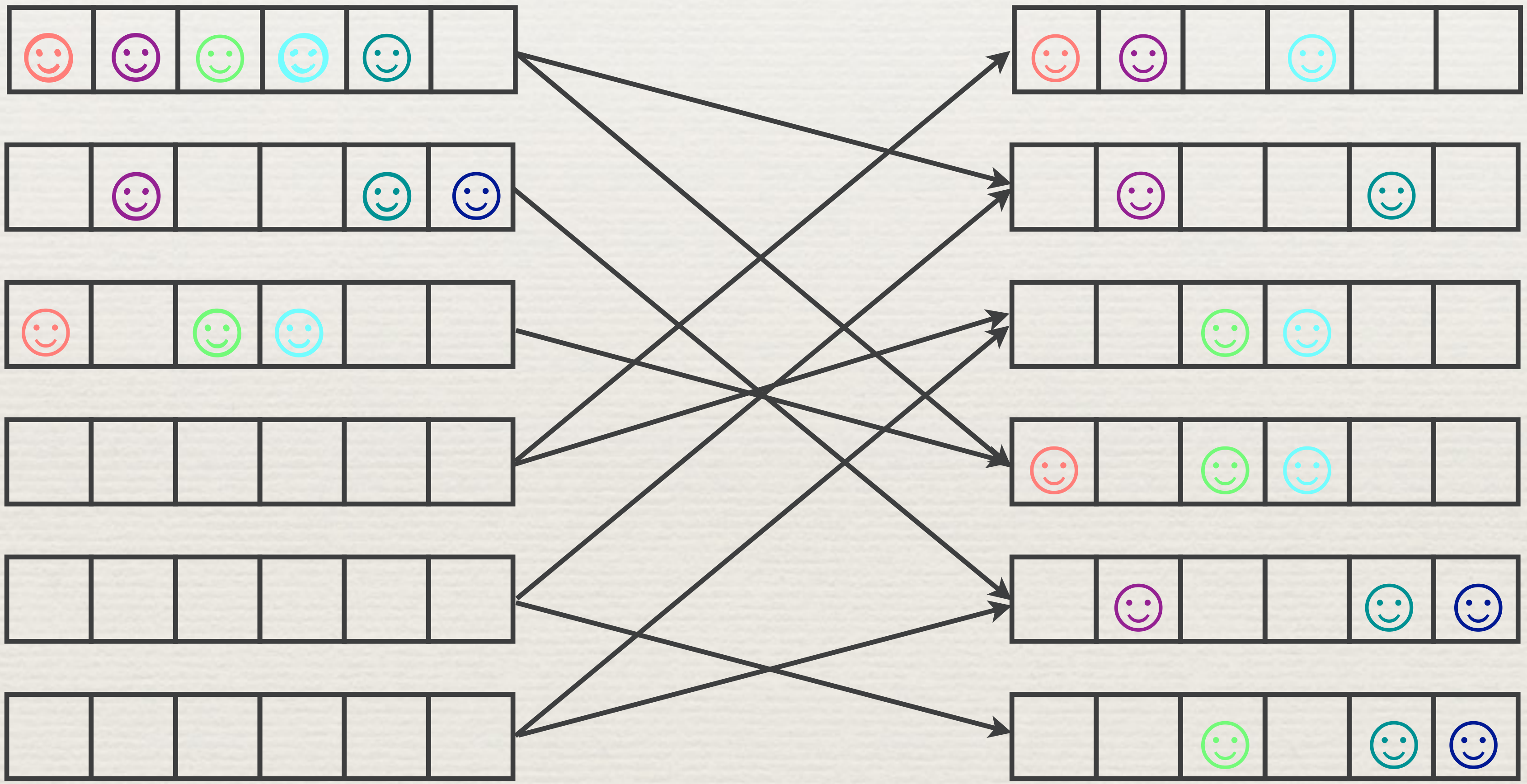

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- We use *broadword programming* to combine HyperLogLog counters quickly!

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- **Important:** the counter of stream AB is simply

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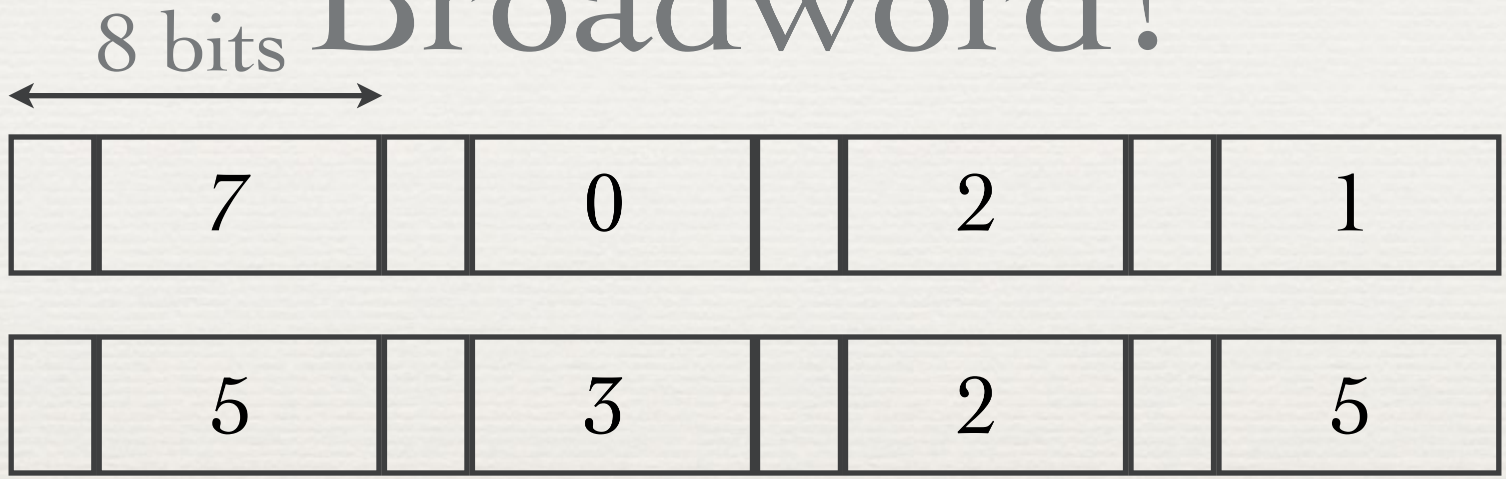
Broadword!



Broadword!

8 bits

1	7	1	0	1	2	1	1
0	5	0	3	0	2	0	5

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$-$

0	5	0	3	0	2	0	5

$=$

8 bits Broadword!

1	7	1	0	1	2	1	1

−

0	5	0	3	0	2	0	5

=

	2		125		0		124

Broadword!

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1	2	0	125	1	0	0	124

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			-				
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			−				
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			=				
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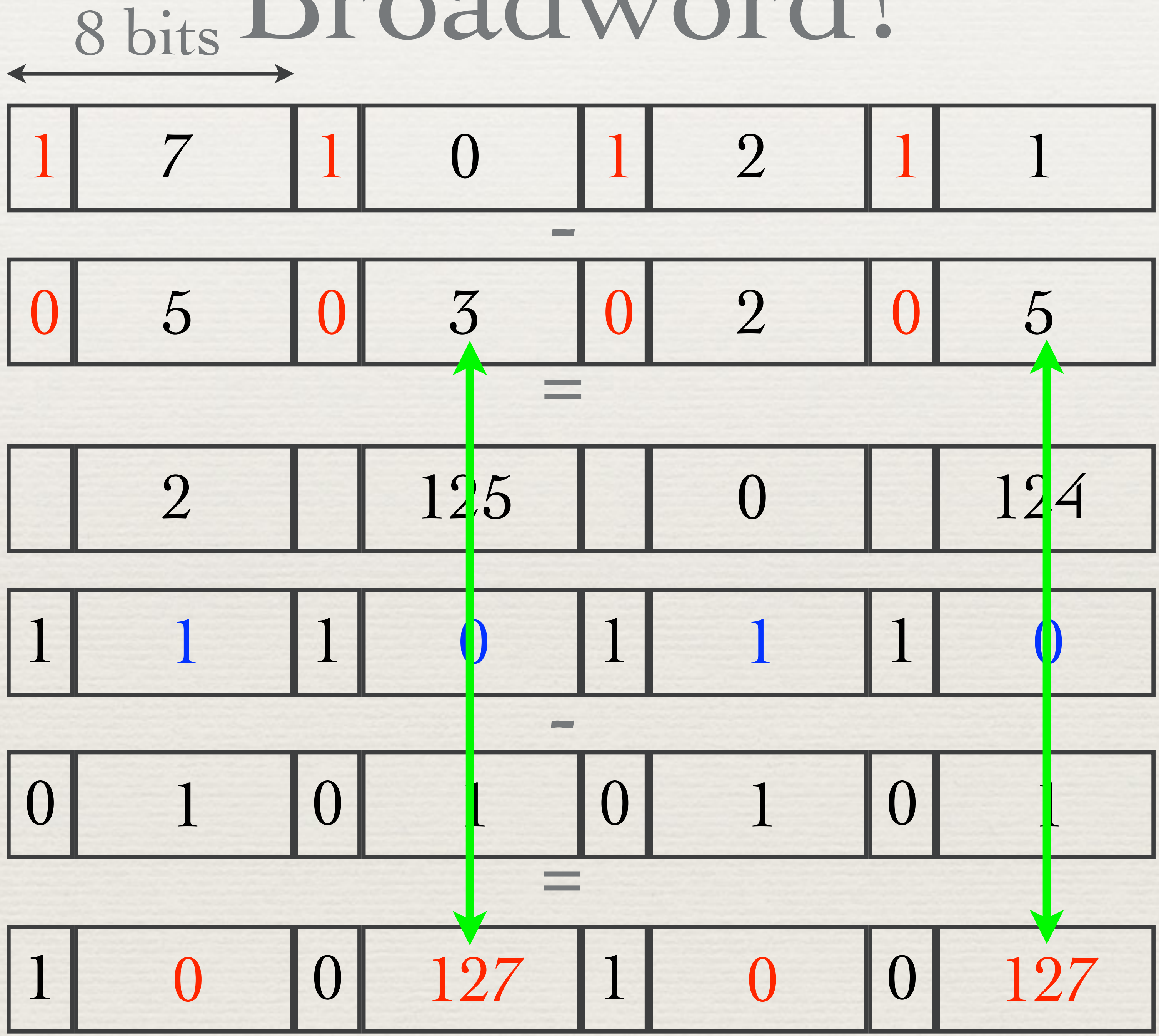
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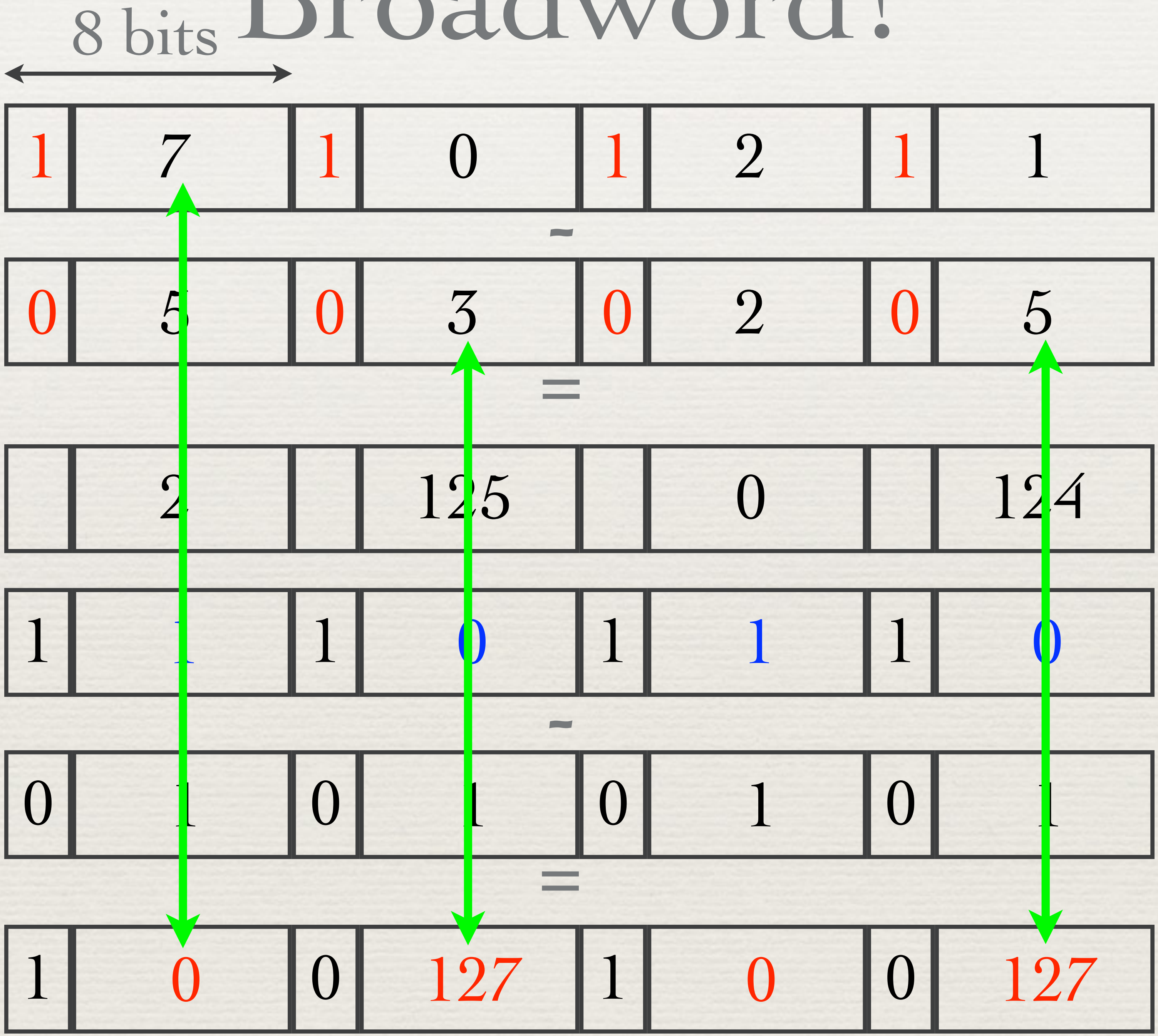
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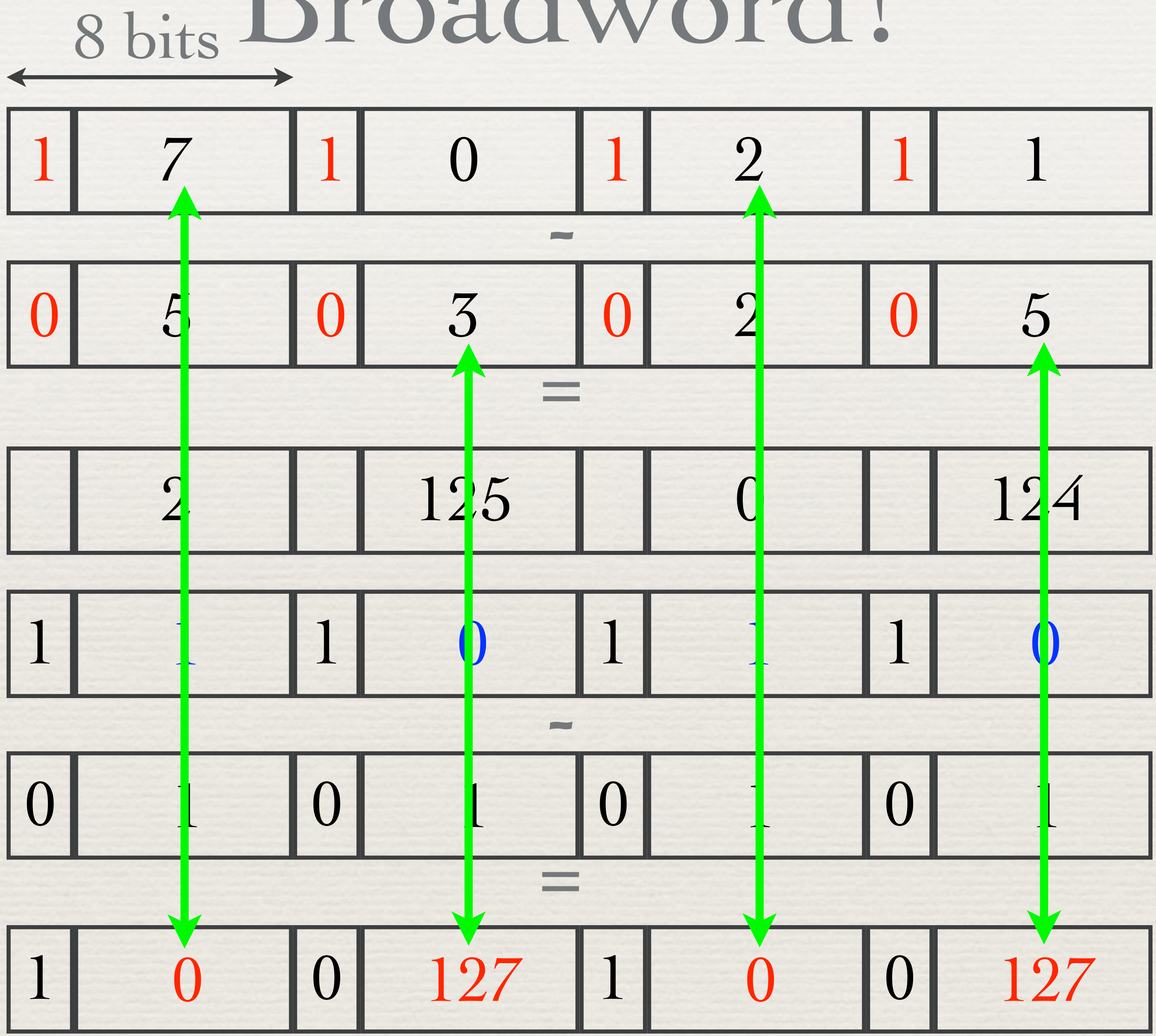
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- We keep track of modifications: we do not maximize with unmodified counters
- Systolic computation: each modified set *signals* back to predecessors that something is going to happen (much fewer updates — $O(m \log n)$ in expectation! [Cohen])
- Multicore exploitation by decomposition: a task is updating just a batch of counters whose overall outdegree is predicted using an Elias-Fano representation of the cumulative outdegree distribution (almost

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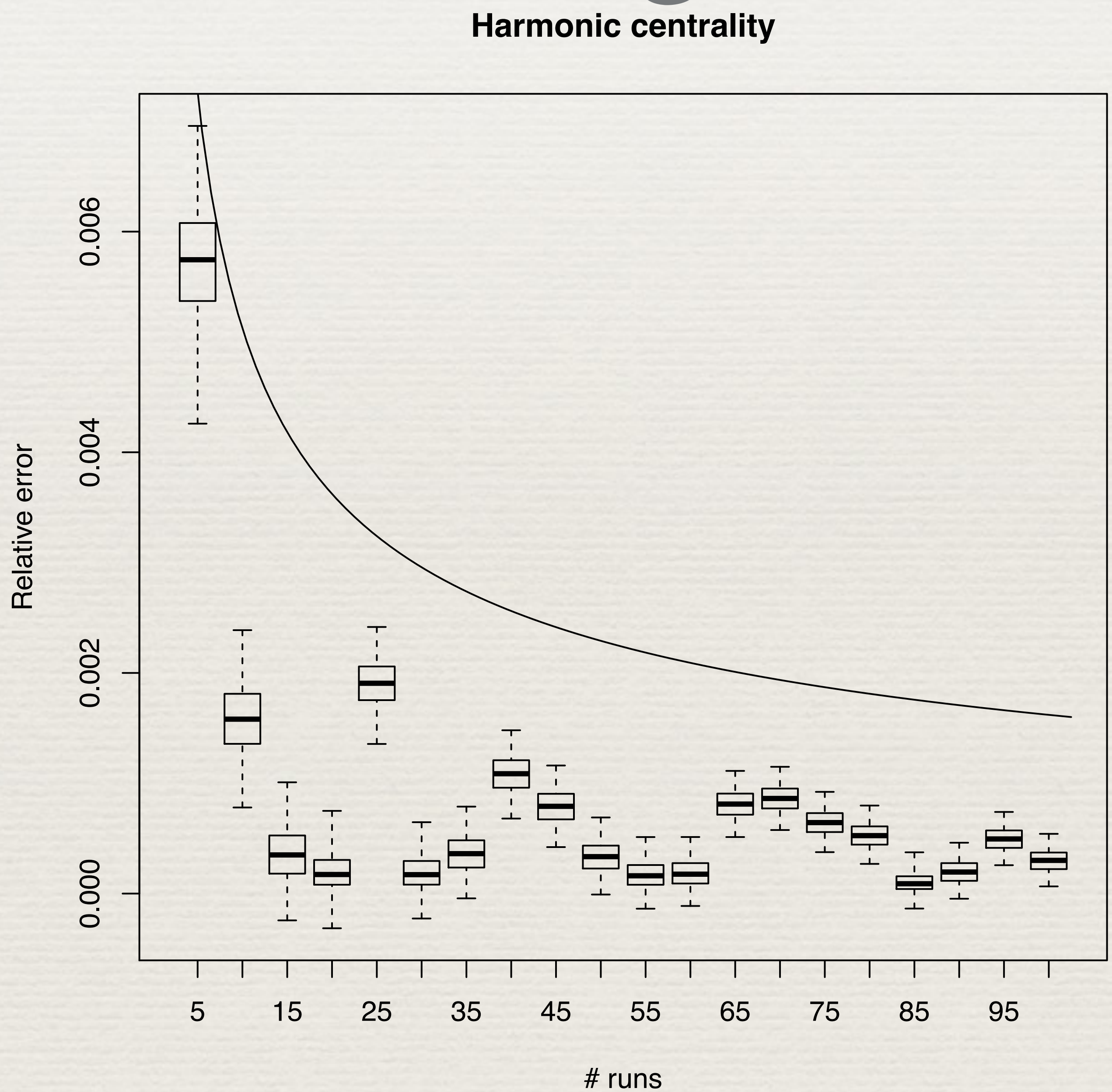
Convergence



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- ...but we can retrofit Cohen's estimators on HyperANF, obtaining an extremely efficient version of Cohen's framework!

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